Acoustics and Fourier Transform

Joshua Weissert*
Northeastern University
(Dated: March 26, 2024)

To analyze sound waves, Fourier Transforms can be used which use a mathematical formula to determine the frequency make up of a wave. Sound outputs from a function generator, tuning forks, a whistle, the human voice, a guitar, a keyboard, and a cigar box guitar were analyzed using MATLAB. The frequency make up of sine and square waves were studied. The acoustic "Beat" phenomenon was created using tuning forks and analyzed. The fast fourier transforms of a whistle and the "aww" vowel were looked at and different musical instruments harmonics were found. The relative intensities for each harmonics were plotted which gave insight into why different instruments have unique sounds. The guitar had higher intensities at lower harmonic levels, the cigar-box guitar had higher intensities at middle harmonics, and the piano had relatively high intensities at higher harmonics.

INTRODUCTION

Sound, a fundamental aspect of our auditory perception, holds captivating properties that can be analyzed and understood through the principles of acoustics. The study of harmonics provides invaluable insights into the intricate composition of sound waves, offering a gateway to deciphering their unique characteristics and applications. Harmonics are multiples of the fundamental frequency of a sound. If a 200 Hz signal is being played, its second harmonic occurs at 400 Hz.

At the core of this investigation lies the powerful mathematical tool known as the Fourier Transform. Developed by Jean-Baptiste Joseph Fourier, this transformative concept allows us to dissect complex sound signals and extract their constituent frequency components. By employing the Fourier Transform, we can unravel the harmonics concealed within various sounds, revealing their individual contributions and forming a comprehensive understanding of their complex structures. Modern communication using Fast Fourier Transforms (FFTs) to analyze and transmit signals efficiently.

In this experiment, the audio outputs and FFTs of different sources will be analyzed and compared. This includes a function generator, tuning forks, whistles, the human voice, and musical instruments.

APPARATUS

The apparatus consisted of the following.

- GW Instek Function Generator, GFG 8219A
- Tektronik Digital Oscilloscope, TBS 1052B
- Tuning Forks of various frequencies
- Custom Made Audio Preamiplifier
- Digital Piano
- Guitar
- Microphone
- Speaker

PROCEDURES AND RESULTS

A. Basic FFT

The sine wave, $y = sin(2 * \pi * 100 * x)$ from x = 0 to 0.01 seconds, using 2048 points, was plotted in MATLAB. Using a Hamming Filter, the FFT was computed and plotted in the region of interest near 100 Hz. This can be seen in Figure 1. The frequency, f_o, at the peak of the FFT is 100 Hz. The FWHM (full-width at half-maximum) linewidth, Γ was 20 Hz and the full range of frequency Δf was 40 Hz.

The sine wave was then plotted from 0 to 1.0 seconds with the same number of points. The FFT was calculated and can be seen in Figure 2. The new Γ was 2 Hz with a full range of frequency of 4 Hz. The smaller time range allowed for 10 cycles to be plotted. The longer time range plotted 100 cycles. This shows that the number of plotted cycles is inversely proportional to the linewidth of the FFT peak. It is also inversely proportional to the full frequency range. High precision FFTs require a high amount of collected cycles. A large frequency range FFT would require a short time collection.

B. Sine and Square Waves

The next part of the experiment was to compare a sine wave and a square wave and their FFTs. A function generator was used to output a 1 kHz sine wave. The generator was hooked up to the oscilloscope which was scaled to see the full wave. The sine wave and its FFT can be seen in Figure 4.

This was then repeated with a 1 kHz square wave. The results can be seen in Figure 5.

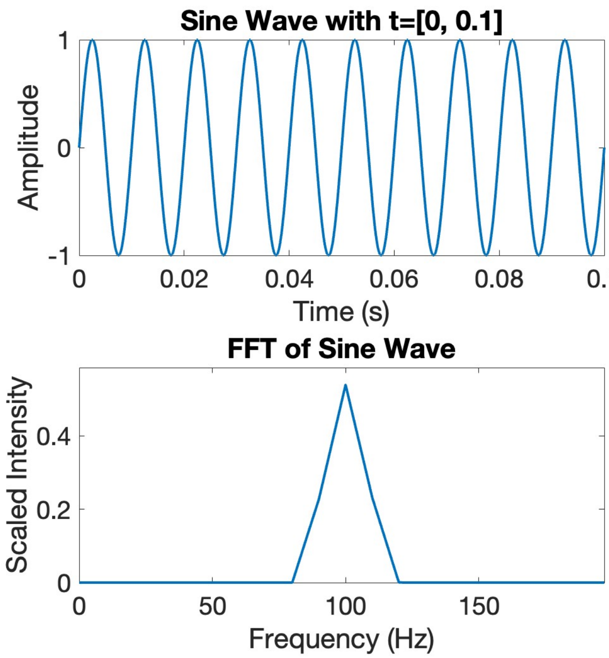

FIG. 1. Sine Wave with frequency of 100 Hz with time axis from 0 to 0.1 and its corresponding FFT

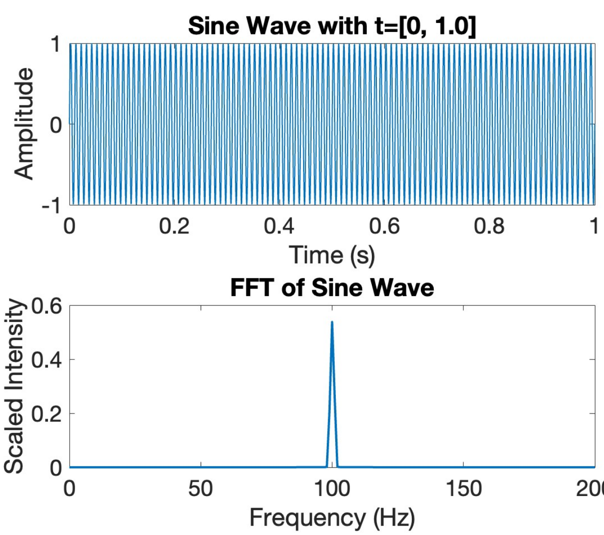

FIG. 2. Sine Wave with frequency of 100 Hz with time axis from 0 to 1.0 and its corresponding FFT

It is shown that a sine wave has a single frequency peak but a square wave is made up of many harmonic frequencies with decreasing amplitudes as frequency increases. The main peaks of both waves occur at 1 kHz but the square wave has harmonics at odd numbered harmonics (3 kHz, 5 kHz, 7 kHz, 9 kHz, etc).

The function generator was then hooked up to a speaker to listen to the differences between the sine and

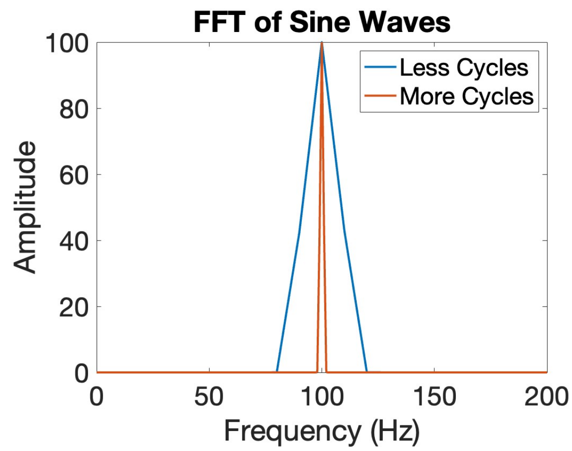

FIG. 3. FFT of both sine waves scaled to an amplitude of 100

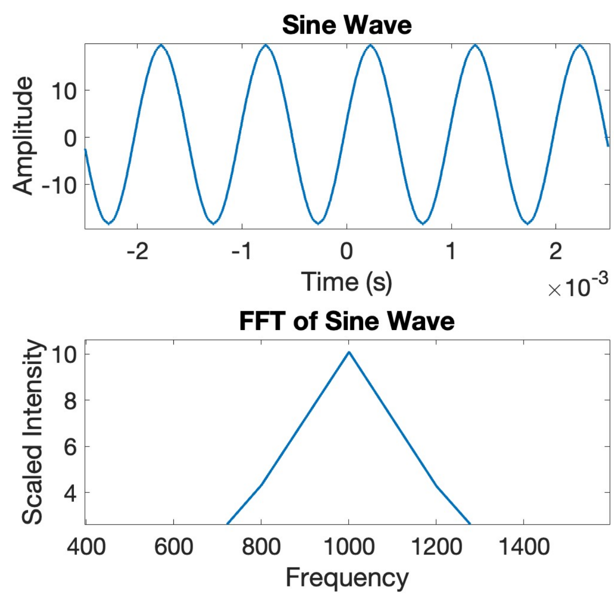

FIG. 4. Sine wave and its corresponding FFT

square wave. The sine wave was loud and sharp compared to the square wave which was softer and sounded more muted. This is due to the added harmonics of the square wave which adds a smoothing affect to human ears.

C. Tuning Forks

The next part of the experiment was to use tuning forks to observe a beat phenomenon. When two sound waves with slightly different frequencies interact, the adding of the two waves results in a beat frequency. Two tuning forks with a frequency rating of 512 Hz were used. The oscilloscope was connected to a microphone. The

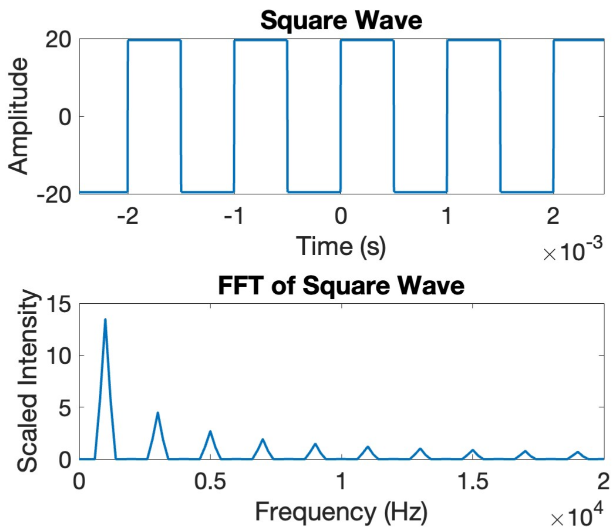

FIG. 5. Square wave and its corresponding FFT

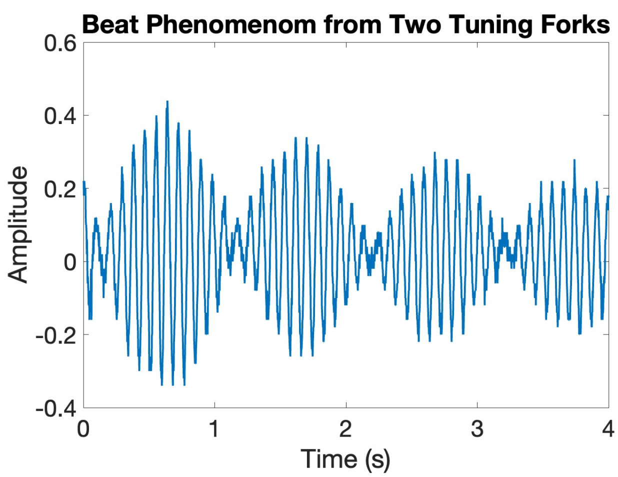

FIG. 6. Audio output of two tuning forks with the same rated frequency

tuning forks were played simultaneously into the microphone and the waveform was captured. This can be seen in Figure 6. The graph has two noticable frequencies. The lower frequency is the beat frequency. Even though the tuning forks were rated for the same frequency they will inherently have slightly different frequencies from the manufacturing process. This difference in frequencies can be seen in the figure and results in a beat frequency of $0.97 \pm 0.02\, Hz$. This was calculated by taking the inverse of the beat period found from the graph.

D. Whistle

A whistle from two different people were then recorded using the microphone setup. Both people attempted to whistle at the same pitch and intensity. Each whistles waveform and FFT can be seen in Figure 7 and Figure 8. As seen in these figures, both FFTs have a sharp peak at a single frequency and no obvious harmonics. The whistles from the two different people are not very distinguishable except for a little bit of noise in Person 1's whistling. The physiology of whistling can be explained by comparing the mouth to a Helmholtz resonator which is a rigid container with a small hole at one end and a larger one to emit sound at the other end. The lips acts as one bound for this resonator and the raised tongue acts as another [1].

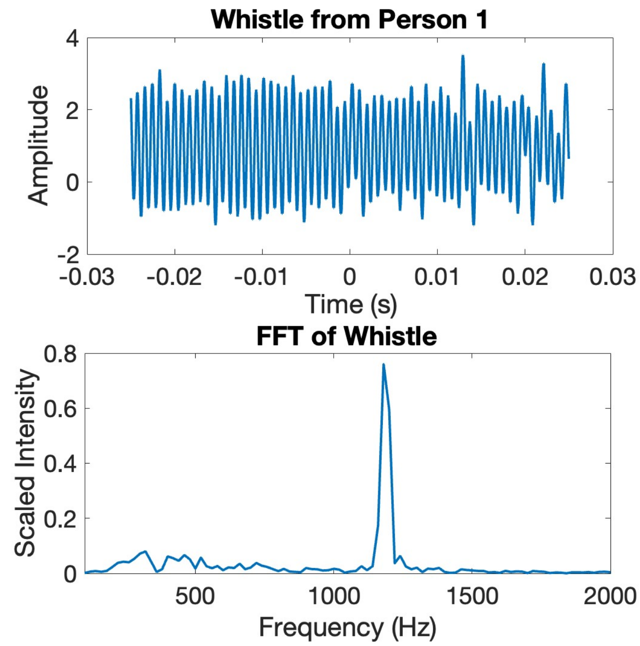

FIG. 7. Whistle from Person 1 audio output and FFT

E. Human Voice

The next part of the experiment was to analyze the human voice. One person said the vowel ("aww") into the microphone. The waveform and its FFT can be seen in Figure 9. The voice is shown to have a fundamental frequency of around 130 Hz with upper harmonics occurring at 260 Hz, 390 Hz, 530 Hz, 660 Hz, 790 Hz, 920 Hz, and multiple other values. The quality of a persons voice will greatly affect the FFT of its waveform. The physical structure of someones vocal cavity and the way they hold their pitch will affect the harmonics of their voice and result in different sounding voices to human ears.

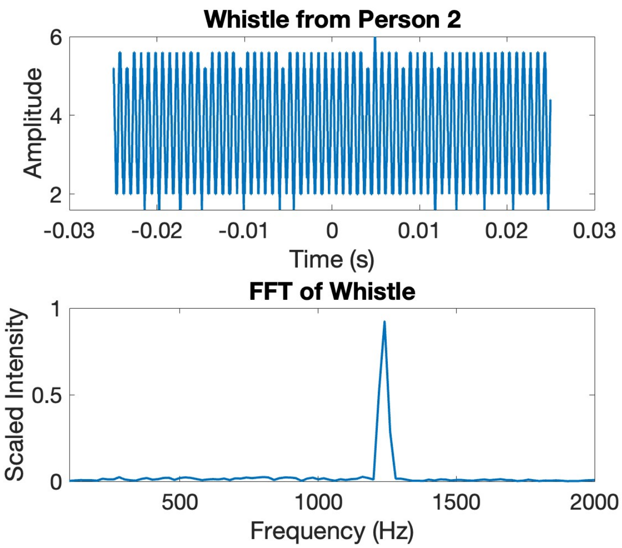

FIG. 8. Whistle from Person 2 audio output and FFT

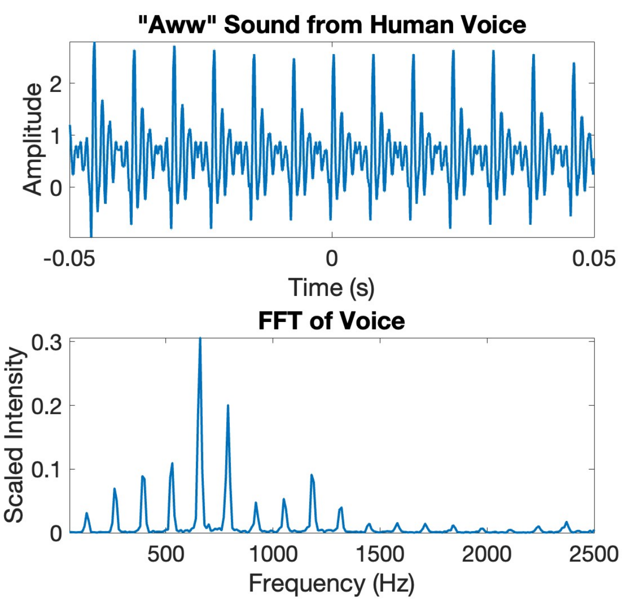

FIG. 9. Audio output from a human voice saying "Aww" and its FFT

F. Musical Harmonics

The next part of the investigation was to analyze the sound outputs from musical instruments. A guitar and an electrical keyboard were used. A G3 (f $\approx$ 196 Hz) was played into the microphone on both instruments. For the guitar, it is important to note that the microphone was placed next to the cavity in the body of the guitar as this is where the sound is amplified. The two audio outputs and their FFTs can be seen in Figure 10 and Figure 11. A plot of both FFTs can be seen in Figure

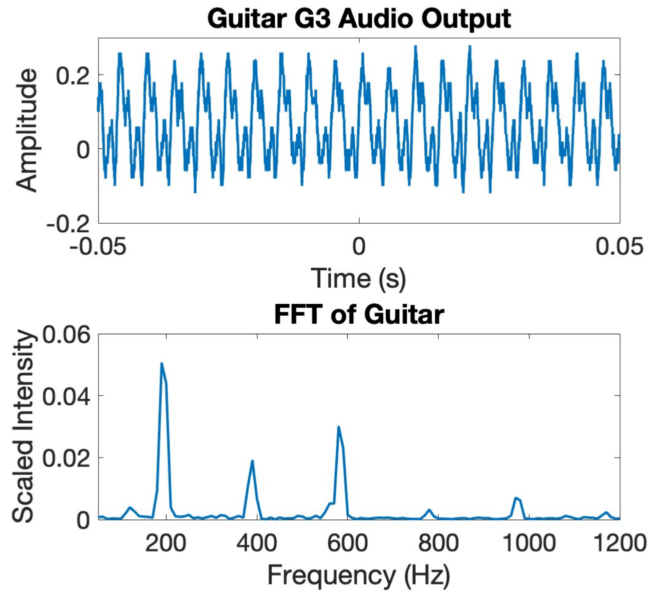

FIG. 10. Guitar playing G3 audio output and its FFT

12. It can be seen that the guitar has two fundamental frequencies. One at around 190 Hz and the other at 120 Hz. It then has multiple harmonics of the 190 Hz peak. The piano shares many of these harmonics but also has much higher intensities at higher harmonics. A plot of the relative amplitudes of the harmonics for each instrument can be seen in Figure 13. The graph was scaled so the maximum value for each instrument was 100. It is shown that the piano has more prominent higher harmonics than the guitar. This difference in harmonic expression is what results in different sounds from the two instruments.

G. Optional

This part of the experiment shows why sampling rates matter in the process of FFTs. The function $y = sin(2 * \pi * 100 * x)$ was graphed from x values of 0 to 0.1 using 100 points and then using only 10 points. The result can be seen in Figure 14. Even though the frequency of the function is 100 Hz, the two graphs seem to show different frequencies. This is because of under sampling with the 10 points case. When data doesn't have a high enough sample rate it can inaccurately represent the data. To avoid this, the Nyquist sampling frequency theorem must be used. This states that the sampling frequency should be at least two times the maximum frequency of the sample to avoid under sampling.

For the last part of the investigation, a cigar-box guitar was brought in. This instrument gives off a sound similar to a normal acoustic guitar but is more twangy and can

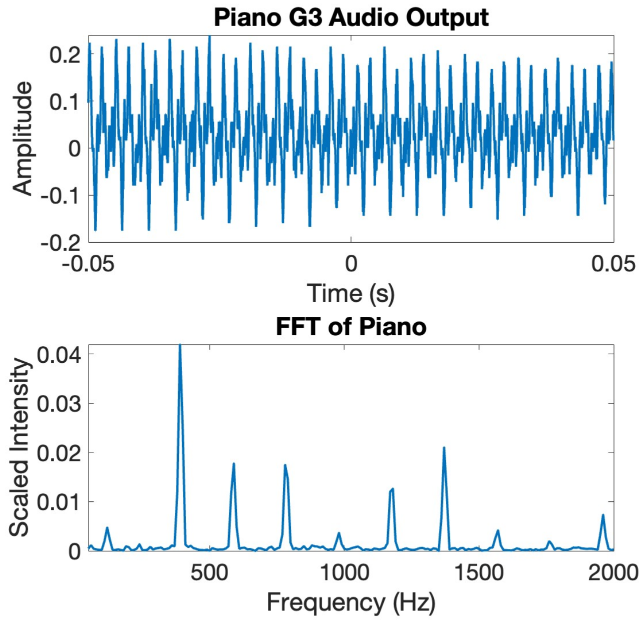

FIG. 11. Piano playing G3 audio output and its FFT

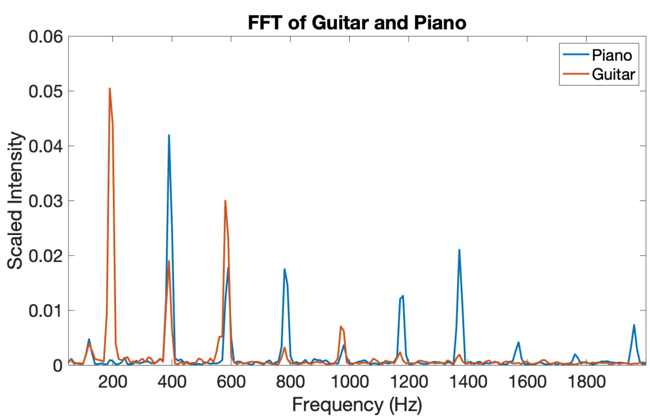

FIG. 12. Comparison of FFTs for Guitar and Piano playing the same note

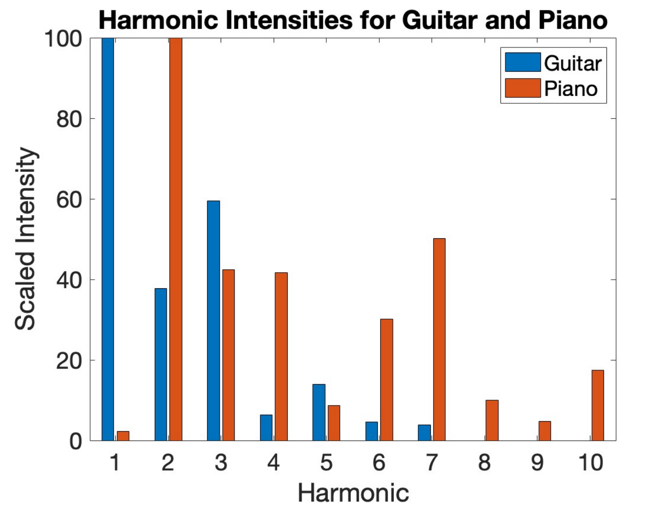

FIG. 13. Comparison of Harmonics and corresponding intensities for Guitar and Piano playing the same note

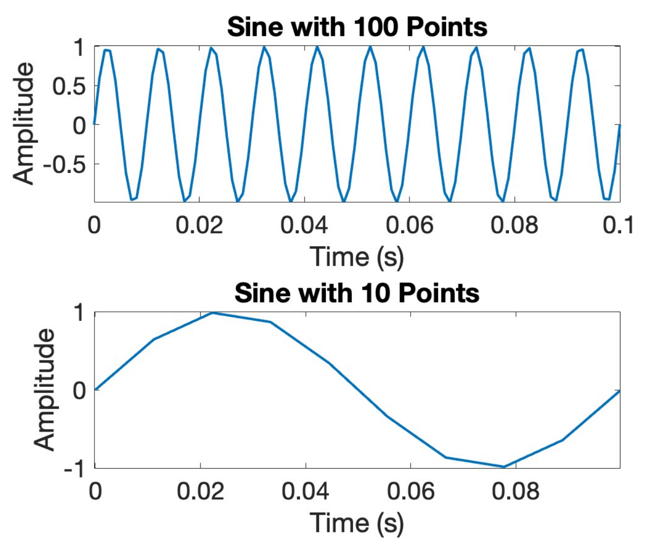

FIG. 14. Comparison of a sine wave using 100 points and 10 points

be used more in a folk music setting. The audio output and FFT of the cigar box guitar can be seen in Figure 15. The harmonics and relative amplitudes are compared to the other instruments in Figure 16.

This shows that the cigar-box guitar used different harmonic intensities than both the normal guitar and the piano. The normal guitar has higher intensities on lower harmonics (1-3), the cigar box guitar has higher intensities on middles harmonics (3-6) and the piano shows high intensities in higher harmonics (7-10). This explains the twangy nature of the cigar box guitar sound as higher harmonics can lead to that.

SUMMARY

This experiment studied Fourier Transforms by using audio outputs from various different sources. The FFT of a simple sine wave was analyzed to show that linewidth and full frequency range of an FFT decreases as the amount of cycles acquired increases.

Then, a function generator was used to study the difference between a sine and a square wave of the same frequency. This showed that the sine wave has a single spike on its FFT, meaning there is one frequency in the wave. However, the square wave had the same fundamental frequency but also had odd numbered harmonics

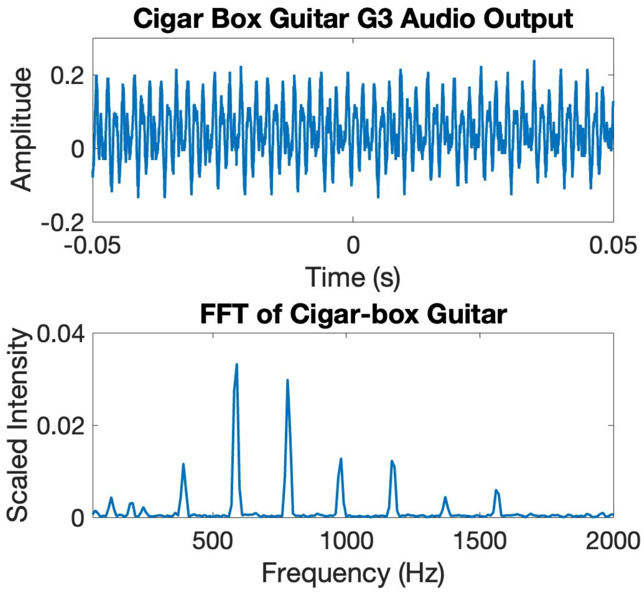

FIG. 15. Cigar-Box Guitar playing G3 audio output and its FFT

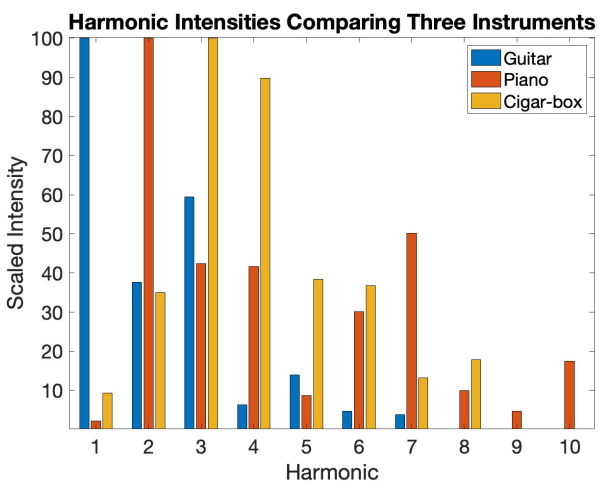

FIG. 16. Comparison of Harmonics and corresponding intensities for Guitar, Piano, and Cigar-box Guitar playing the same note

that decreased in intensity the higher they got.

Next, two tuning forks with the same rated frequency output were played at the same time to achieve a beat phenomenon. This phenomenon occured due to the tuning forks outputting slightly different frequencies. The sound waves from each interfered with eachother to get the resulting "beat".

A whistle sound was then studied from two different people. Each person tried to whistle at the same pitch and intensity and this showed that a whistle does not have any harmonics. Both whistles were also very similar to each other.

The human voice was then studied. A person said the "aww" vowel into the microphone and the FFT of the audio showed multiple harmonics with the most intense being the 5th harmonic.

Three different musical instruments were then compared. A normal acoustic guitar, an electric keyboard, and a cigar box guitar were all recorded playing a G3 ($f \approx 196$). The relative intensities of each harmonic were graphed to show that each instrument sounds different from each other because of the harmonics that are prominent. The normal guitar has higher intensities for low harmonics compared to the cigar box guitar which utilizes mid harmonics and the piano which has the highest harmonics of the three. All three instruments had a fundamental frequency around 200 Hz with a second fundamental frequency around 120 Hz. The only instrument that had the highest intensity at its fundamental frequency was the normal acoustic guitar.

* weissert.j@northeastern.edu

[1] "The physiology of oral whistling: a combined radiographic and MRI analysis," *Azola et all,*

* weissert.j@northeastern.edu