From: Nima Rakhshandeh Tali, Samuel Scroggie, Joshua Weissert, Jacob Ward, Sean Isaac, Christopher Zhang

Company Name: Java Tech

Date: June 22, 2023

To: Prof. Hamid Nayeb-Hashemi & Evan Emanuel

Re: ME4550 Final Portfolio – Coffee Sorting Machine for Micro-Mills in Costa Rica

1 Executive Summary

Micro-mills, defined as coffee-growing operations that harvest 100-500 kg of coffee beans per year, are prevalent in Costa Rica. These businesses must sort their product by size to access lucrative international markets. However, current commercial solutions are too large and expensive for the operating scale of micro-mills. To fill this need, we at Java Tech have designed a machine that can sort coffee beans by size at an affordable price and appropriate scale for Costa Rican micro-mills. The original proposal attached here outlines the intended design and associated specifications. After one major revision of replacing an Archimedes screw with a power screw as the primary driving mechanism of the sorting process, calculations were done for components, attachments, and a power train. Regarding components, the frame was analyzed for safety against yielding, buckling, and fatigue failure modes to determine a suitable material and cross section for the constituent members. We determined that 1006 HR steel, the cheapest extruded steel available on the market, may be used for most components. Three connections of interest were then analyzed to ensure that the components may be joined safely and would not detach during operation. Finally, an experiment was devised to quantify the torque that must be provided to the power screw by the power train to successfully sort the beans. Based on the desired sorting speed and associated power requirement, a motor was selected from McMaster-Carr and a gearbox was designed to achieve the appropriate speed and torque outputs.

We at Java Tech believe this machine would be a massive benefit for micro-mills as it is tailor-made for their specific needs and budget range. We hope that the rigorous design process outlined in the following memos may be used in future work to bring this product to market. Finally, we would like to thank Professor Hashemi for all the help throughout the project.

2 DESIGN PROPOSAL

Problem Statement

Coffee growing micro-mills in Costa Rica need to sort their coffee beans by size to access lucrative international markets. International businesses expect to know the size (including tolerance) of the coffee beans they purchase, so that they can be confident the beans are compatible with their manufacturing processes and/or roasting procedures. However, current mechanized solutions are prohibitively expensive because they are designed for loads relevant to larger, industrial-scale coffee-growing operations. Micro-mills are defined as independent mills that produce finished coffee ready for export in low volume. Micro-mills are typically family-owned farms that no longer must rely on large cooperative mills because they have the equipment to process their own beans [1]. Micro-mills harvest and sort beans once per year, yielding approximately 500 kg of product. Few size-sorting machines appropriate for the scale of micro-mills exist on the market; the proposed design aims to fulfill this need.

Background Research

Existing Products

There are several coffee bean sorting devices available on the market. However, as noted in the problem statement, they are targeted at large scale producers whose production is in a different order of magnitude than that of the micro-mills. High-end solutions optically sort the beans by color in addition to separating them by size; these devices are very large and expensive, such as the one shown below:

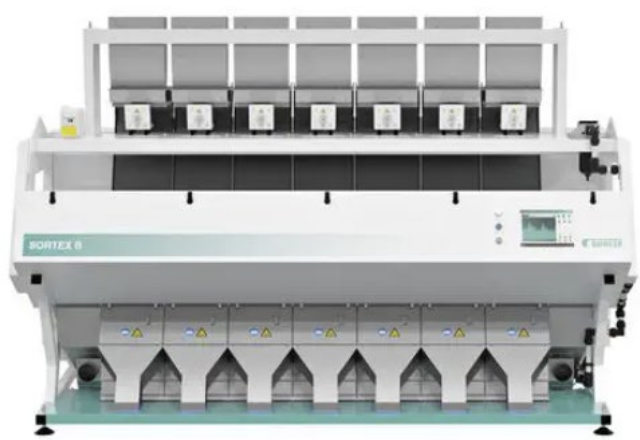


Figure 1 - Industrial Machine with Optical Sorting Capabilities [2]

From an interview with a micro-mill owner in Costa Rica, micro-mills do not require the extensive features provided by these industrial devices. Typically, current commercially available size sorters use a feeding hopper that dispenses the coffee beans onto a flat, perforated platform that forms the sieve. The sieve(s) oscillate back and forth, mimicking the motion of a human doing so manually. A bean falls through to the next sieve if it is smaller than the perforations. One such product, which uses multiple sieve platforms such that the smallest beans are filtered to the bottom platform, is shown below in Figure 2:

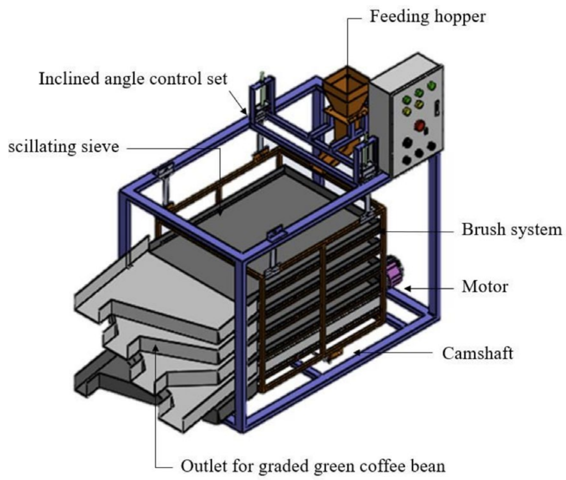


Figure 2 - Example Design with Oscillating Sieve Platforms [3]

Mechanisms Used in Proposed Design

The team proposes that the vibration method employed by the design in Figure 2 puts the device under many cycles of stress, making it susceptible to general wear, and eventually experience fatigue failure if not designed properly. To combat this, the team suggests a device that uses rotary motion to perform the sorting rather than lateral vibration. The inspiration for the mechanism comes from cement mixer trucks, which feature a large drum with an Archimedes screw pattern on the inside, as shown below:

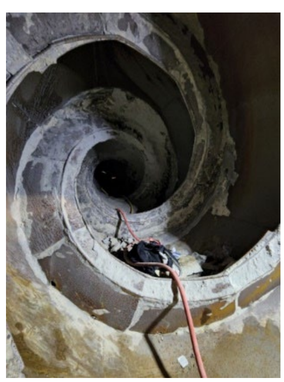


Figure 3 - Interior of A Cement Mixer Truck [4]

The Archimedes screw pulls the beans along increasingly wide slits in the drum, which filters the beans by size. When a bean runs over a slit slightly larger than its width, it falls through into a bin. Further research into the design of Archimedes screw mechanisms will be necessary to ensure smooth integration into the coffee sorting device. The proposed design will borrow the feeding hopper, motor/gearbox control, and overall structural frame from the device in Figure 2, incorporating some modifications and substituting the Archimedes screw working principle.

Overview

A concept sketch showing the proposed design targeted at micro-mills is shown below.

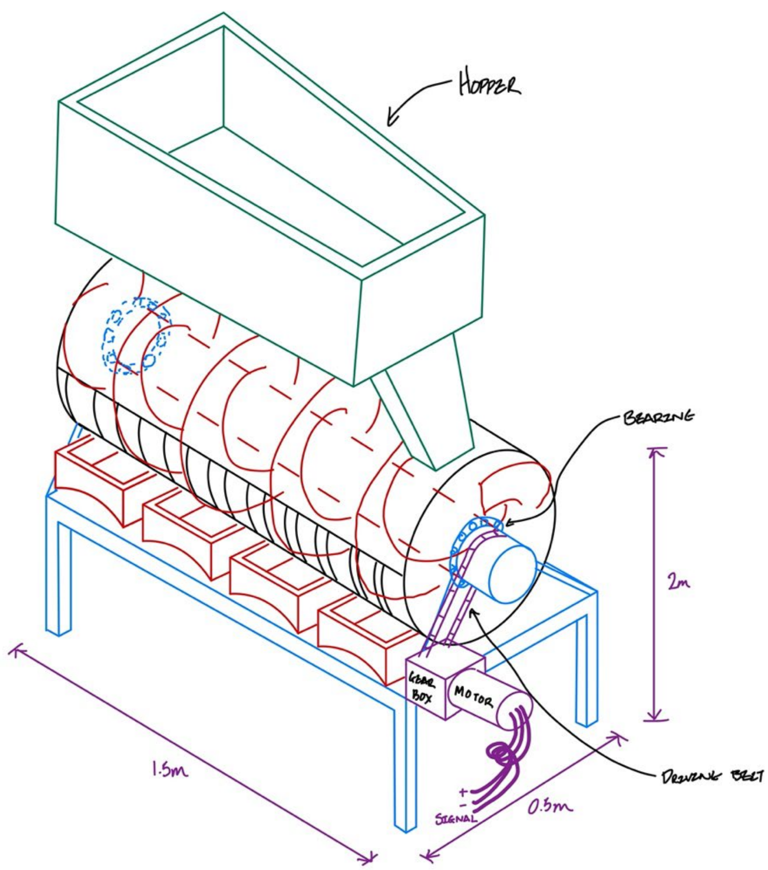


Figure 4 - Concept Sketch of Proposed Design

The beans are fed into a hopper, which then feeds them into the sieve "drum". This setup allows the operator to load all the beans at once rather than having to continuously feed the hopper. Beans waiting in the hopper are dispensed at a constant flow rate into the sieve drum.

The sieve portion consists of a large, fixed drum with slits of increasing width on the bottom face to sort the beans. There is a main shaft running through the center of the drum, which will have a Archimedes screw (helical surface surrounding a central cylindrical shaft) pattern along the length, as shown in Figure 5 below:

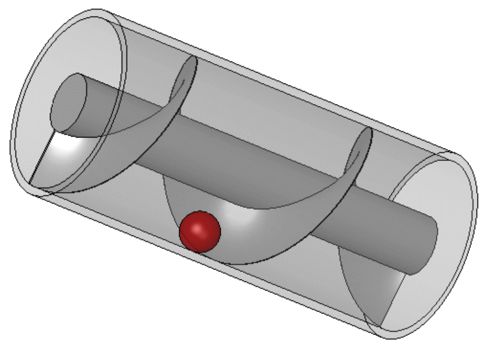


Figure 5 5- Archimedes Screw and Drum Example [5]

The Archimedes screw will be suspended by low-friction ball bearings on either end and rotated by a motor, gearbox, and belt powertrain. This design allows the beans to move forward linearly as the shaft rotates, and the drum remains fixed in place. The size of the slits increases along the length of the sieve, such that the smallest beans are filtered out near the hopper. The containers are to be placed on the tabletop to enable the mill employees to remove them without bending down.

Frame and Main Components

The frame should be constructed from a strong, durable material that is resistant to the high humidity and heat in Costa Rica, the target market. Options such as 80/20 extruded aluminum will be explored due to its versatility and relatively low cost. Alloy steels, subject to different heat treatments depending on the final load conditions, will also be explored. The critical areas for analysis are the table legs and tabletop surface, which will be subject to cyclic loading. The weight of the beans is applied to and removed from the tabletop, so fatigue failure must also be considered. The legs must be designed so they do not buckle under the load. A mixture of materials may be used if appropriate, with the goal of maintaining a lightweight device to reduce shipping costs.

Other critical areas of analysis are the attachment points of the hopper/trough to the frame, which will also be subject to the heavy weight of the beans, and the bearings which support the

Archimedes screw. The shaft itself must be strong enough to avoid yielding under the torsional and bending stresses associated with driving the beans forward.

The design of the drum may need to be refined in future work. The major area of concern with the current design is the analysis of stress concentrations on the drum, which has many holes on the bottom. If this analysis becomes too complicated with hand calculations and FEA becomes necessary, this design will need to be revisited. Other possible options include a linear conveyor-belt style sieve mechanism, using rollers or a power screw that pushes all the beans toward slits of different sizes in the tabletop surface. There may be simplifying assumptions made to facilitate analysis of the current design. The drum is currently proposed as formed sheet metal, welded along the seam. Other options that do not require a weld may be considered.

Attachment of Components

Currently, the structural elements of the frame will be bolted together. Gusseted brackets may be incorporated to increase rigidity if necessary. As noted above, the shaft will be supported by bearings on either end; the exact method of attachment to the frame will need to be designed. Welding the frame may be explored as an alternative option, but it likely will not be necessary.

Ideally, the sieving drum will consist of a metal drum (cylinder) with slits of increasing size machined out of the bottom half. Alternative manufacturing techniques could involve plasma cutting standard sheet metal and pressing it into a cylinder shape; two circular cut outs would then be welded to either side.

Power Train

A DC brushless motor and a gear train that increases torque will be used in the proposed design. The high torque, lower rpm output will drive the main shaft which holds the Archimedes screw. Speed is not a critical factor, as micro-mills only harvest and sort their beans once per year. Thus, the driving factors for the powertrain will be high torque to effectively move the beans and low overall cost. Gears must be selected such that they can withstand the large forces required for the sorting process.

Design Specifications

Overall Design Parameters

- Capable of holding and sorting up to 100 kg of coffee beans at a time. For a maximum harvest of 500 kg per year, this would require loading the machine five times.

- Budget-friendly, < $2,000
- Overall footprint of approximately 1.5 m long × 0.5 m wide × 2 m tall
    - Tabletop surface located 1 m off the ground for both easy loading on beans into the hopper, and retrieval of bean bins once sorted by size.
- Lightweight
- Long life – since the machine theoretically only cycles five times per year at the maximum harvest yield, it should last at least as long as the business.
- Requires minimal maintenance and supervision during operation
- Safe, easily operated by able-bodied mill employees
- Speed not critical, but ideally completes sorting each 100 kg batch within 1 hour
- Easily cleanable

Component Design Parameters

Structural Frame

- Strong, rigid to avoid large deformations
- Resistant to fatigue and buckling failure modes
- Resistant to humidity and other environmental factors
    - Good wear properties

Drum

- Lightweight
- Resists deformation under load of beans
- Welds withstand weight of beans

Shaft/Archimedes Screw

- High yield strength
- Good wear properties
- Long life

Motor + Gearbox

- High torque (to spin drum and beans)
- Gears have good wear properties, teeth withstand cracking/erosion over the life cycle of the device
- Reliable, long life

Potential Materials

Many of the components share similar requirements regarding material properties. In general, materials used in this design must be:

- Strong
- Resistant to wear
- Manufacturable
- Cost-effective
- As lightweight as possible

Due to the large loads this device must bear and its industrial application, it is anticipated that most of the components will be made of metal. It may be beneficial to allow the mill operators to see inside the drum to monitor the progress of the sorting process. For this reason, a transparent thermoplastic may also be considered for the construction of the drum.

Conclusion

Our product is targeted towards micro mills that need to separate approximately 500 kg of coffee beans once a year. Satisfying the needs of these customers requires attention to several important aspects of the design. The device must be relatively small and much cheaper than the large-scale devices currently on the market. It must be very strong, able to last many years, and operate reliably to minimize capital and operating costs for the mill. To further reduce costs, custom sizes and scarcely available parts should be eliminated wherever possible.

The proposed design to meet these requirements is summarized as follows. An Archimedes screw on a central shaft is used to push and agitate beans that come in from a hopper. The Archimedes screw pushes the beans over a drum with specific sized holes that are cut out to let the beans fall into labeled buckets. The power that rotates the shaft comes from a gear box that takes in high rpm from a DC brushless motor and outputs high torque. High torque is needed to rotate the weight of the screw and push the weight of the coffee beans.

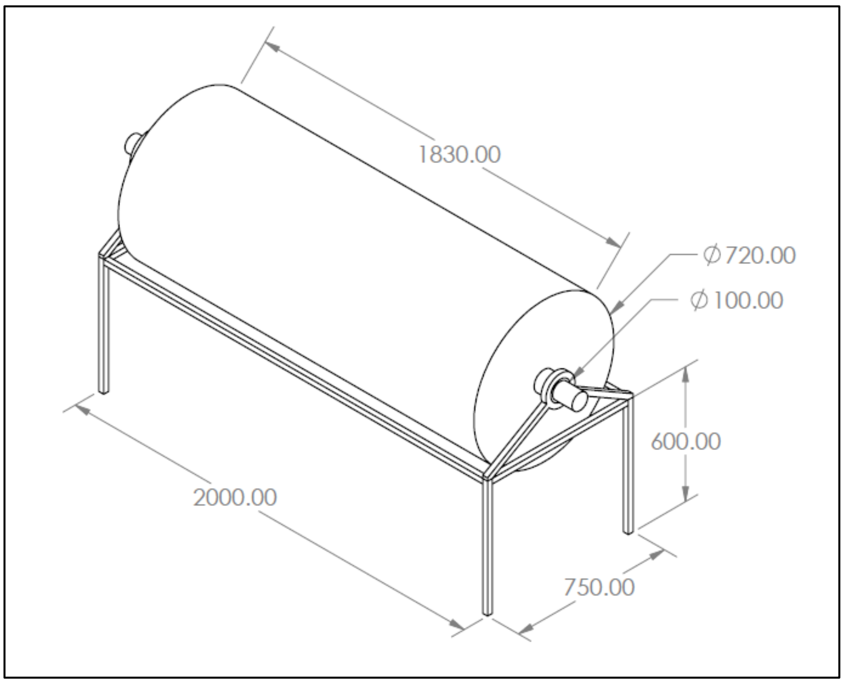
The choice to stay away from a vibratory table sift comes from the downsides of vibrations. Vibrations can be noisy, affect surrounding machinery, and cause stress over time. However, using the proposed mechanism to sort the beans introduces other risks that must be addressed throughout the design process. Primarily, the shaft and drum will be under heavy stress throughout the process, making the material selection and attachment methods of these components critical. The areas of the frame supporting the hopper/trough, bearing the beans waiting to enter the sorting drum, will also be under significant load. It is essential to select materials for these components that satisfy the cost and weight goals while maintaining structural integrity.

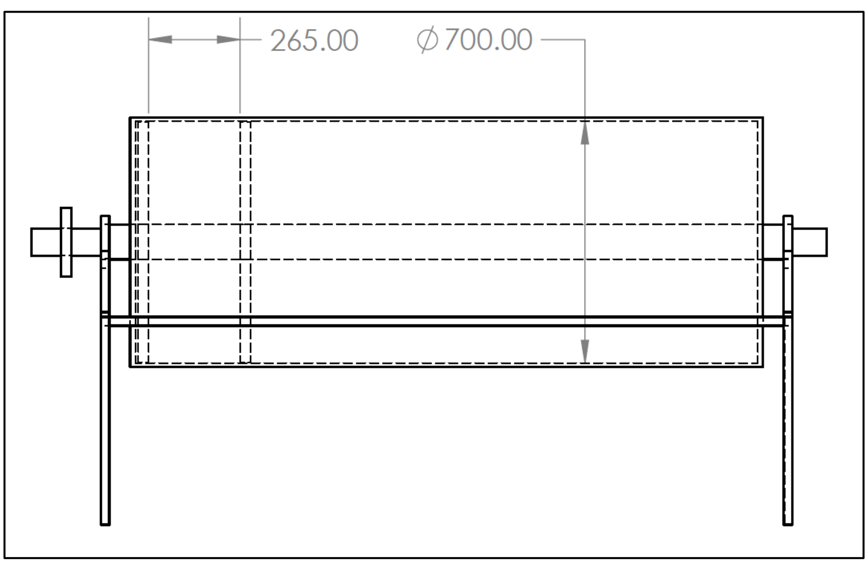
If, for any reason, an element of the proposed design is deemed dangerous or inefficient, alternatives will be promptly investigated. Possible examples include changing the drum composition, shaft mounting scheme, or even using an entirely different sifting mechanism such as spaced-out rollers as used in airport security. The mock-up will help determine potential issues and modifications necessary.

3 COMPONENT DESIGN

Introduction to Components

The original proposed design for the coffee size-sorting device has been modified to simplify construction and facilitate calculations using the techniques learned in class. Rather than using an Archimedes screw to push the coffee beans forward, the central shaft will be a power screw that drives a flat circular plate that moves the beans along the drum. A second plate will be used to keep the beans in a compact cylindrical shape. A CAD model showing the new design and proposed dimensions (all in mm) is shown below:





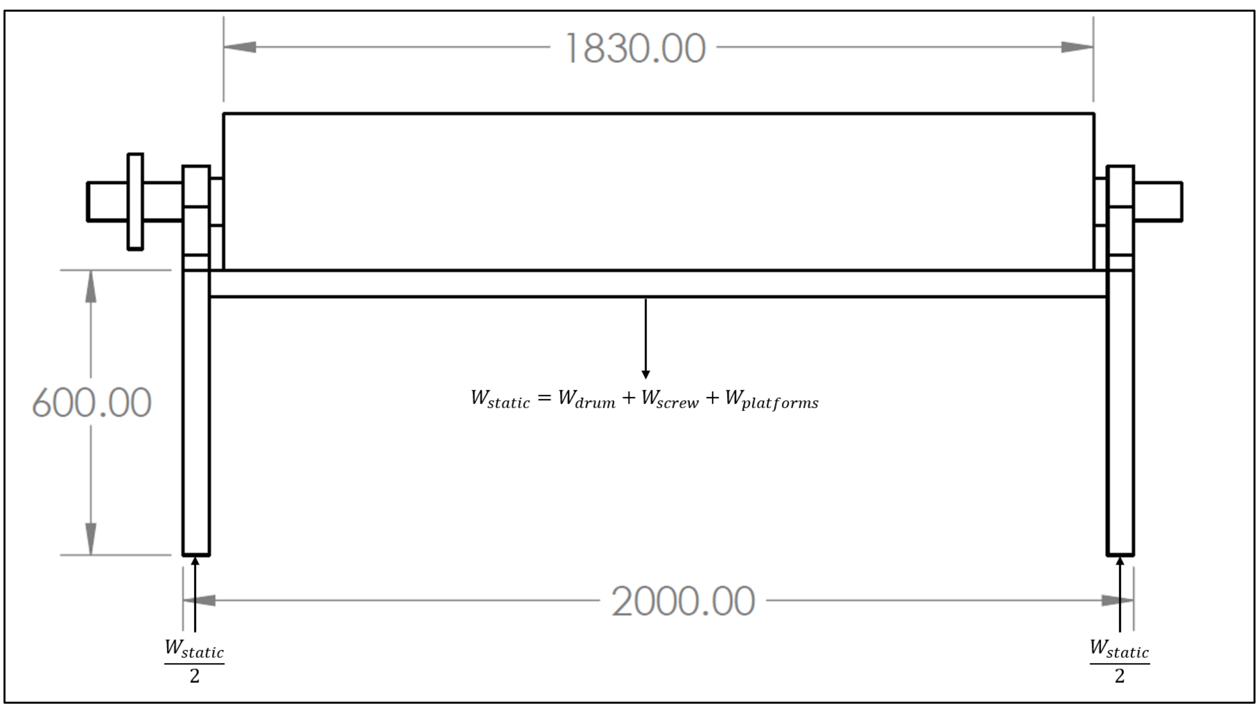
In this memo, the structural frame will be evaluated for safety against static yielding, buckling, and fatigue failure modes. The frame may be modeled as a truss with some members under compression, and all under cyclic loading. The material is selected based on static yielding analysis, and subsequent calculations for buckling and fatigue will verify the material selection. The force required to drive the coffee beans is determined experimentally and used to calculate the torque required on the power screw. Stress analysis of the power screw is subsequently performed.

Loading Conditions & Simplifying Assumptions

By assigning material properties to components, the CAD model shown above was used to evaluate the weights of critical components that the frame will need to support:

Component	Material (general)	Mass (kg)	Weight (N)
Drum	Steel	410.25	4024.6
Power screw	Steel	140	1373
Push Platforms	Steel	176	1726.6
Beans	Coffee	50	735.8

It was assumed that the bearings which support the shaft, drum, and platforms are simply supported, thus meaning that the machine may be taken as a simply supported beam when viewed from the long side under static conditions, as shown below:



Thus, the minimum load supported by each side of the frame is given by:

$$F_{min} = \frac{W_{drum} + W_{screw} + W_{platforms}}{2}$$

These sides will be modeled as trusses composed of pinned two-force members with hollow rectangular cross-sections. The two opposites sides of the frame are subjected to different loading conditions. The "exit" side, where the beans have been fully sorted, is approximated as static loading since the mass of the beans is insignificant at that point. The "feeder" side, where the beans are loaded into the drum, is subject to cyclic loading as the beans are inserted, then moved away and sorted. Thus, the maximum force experienced by the feeder side is given by:

$$F_{max} = F_{min} + W_{beans}$$

From this analysis, the feeder side is subjected to a more significant load and thus will be used for calculation. The truss will be solved for the member forces, which will then be used to analyze safety against yielding, buckling and fatigue.

Part 1: Static Yielding

The yielding analysis for the feeder side of the frame is shown below. The member cross-section was determined by using McMaster-Carr to identify available standard rectangular tubing sizes. We opted for a square section 25 mm × 25 mm on the outside with 2 mm thickness.

PART I: STATIC YIELDING

$W_{drum} = 9.81 \times 410.25 = 4024.6$ N

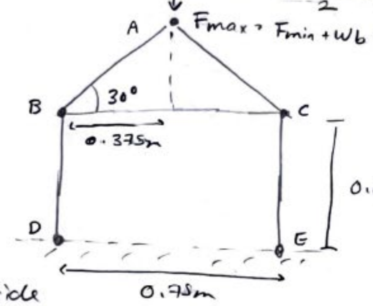
$W_{screw} = 9.81 \times 140 = 1373$ N

$W_{beans} = 9.81 \times 75 = 735.8$ N

$W_{platforms} = 9.81 \times 88 \times 2 = 1726.6$

$F_{min} = \frac{W_d + W_s + W_p}{2} = 3562.3$ N

$F_{max} = F_{min} + W_b = 4298$ N



1) Assume frame simply supports weight of drum & power screw & platforms

2) Worst case scenario when all the beans on the feed side

Joint A

$F_{max} = 4298$ N

Symmetry: $F_{AB} = F_{AC}$

$2F_{AB}\sin 30° = F_{max}$

$F_{AB} = \frac{4298}{2\sin 30°} = \boxed{4298\text{ N}}$

Joint B

$F_{BC} = F_{AB}\cos 30° = \boxed{3722\text{ N}}$

$F_{BD} = F_{AB}\sin 30° = \boxed{2149\text{ N}}$

3) Members AB and AC under most load. Analyze static yielding w/ rectangular tubing:

25 mm; 21 mm; 21 mm; 25 mm

$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{4298}{0.025^2 - 0.021^2} \times 10^{-6} = \boxed{23.36\text{ MPa}}$

$\sigma_{1,2} = \frac{\sigma_{AB}}{2} \pm \sqrt{\left(\frac{\sigma_{AB}}{2}\right)^2 + \tau^2} = 23.36$ MPa, 0, $\sigma_3 = 0$ (plane stress)

MST: $\frac{S_y}{2n} = \frac{\sigma_{pmax} - \sigma_{pmin}}{2} \rightarrow S_y = n(\sigma_{pmax} - \sigma_{pmin})$

4) Design w/ high FS since machine should need minimal inspection.

We'll choose $n = 5$ at first

$S_y = 5(0-(-23.36)) = \boxed{116.8 \text{ MPa}}$

We'll pick up $\boxed{\text{AISI 1006 HR steel}}$, which will give $\boxed{n = \frac{S_y}{\sigma} = \frac{170}{23.36} = 7.28}$ against yielding

Will also need minimum loads for fatigue analysis:

$(F_{AB})_{min} = \frac{3562.3}{2\sin 30°} = 3562.3 \text{ N}$

$(F_{BD})_{min} = 3562.3 \sin 30° = 1781.2 \text{ N}$

$(F_{BC})_{min} = 3562.3 \cos 30° = 3085 \text{ N}$

Based on the worst-case loading conditions, it was determined that AISI 1006 HR steel would provide a very substantial safety factor of $\boxed{n = 7.28}$ against yielding. This is a cheap steel, which is essential to our goal of designing an affordable machine for Costa Rican micro-mills. With this high safety factor, it would likely be possible to make the frame out of aluminum instead, making the device even cheaper. However, we propose that the high safety factor eliminates the need for routine, costly inspections, and enables the potential for infinite fatigue life, as will be shown in Part 3.

Part 2: Buckling

The material and cross-section found in Part 1 were used to perform analysis of buckling of feeder-side members under worst-case loading conditions.

PART II: BUCKLING

From Part I: Members AB, AC, BD, CE in compression.

↳ $(F_{AB})_{max} = (F_{AC})_{max} = 4298 \text{ N}$

$(F_{BD})_{max} = (F_{CE})_{max} = 2149 \text{ N}$

Column Properties

$A = 0.025^2 - 0.021^2 = 1.84 \times 10^{-4} \text{ m}^2$

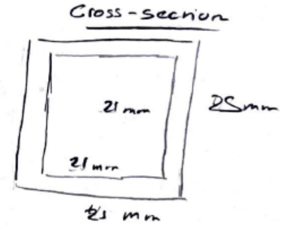
$I = \frac{1}{12}\left[(0.025)(0.025)^3 - (0.021)(0.021)^3\right] = 1.635 \times 10^{-8} \text{ m}^4$

$E = 200 \times 10^9 \text{ N/m}^2$

$l_{AB} = \frac{0.375}{\cos 30°} = 0.433 \text{ m}; \quad l_{BD} = 0.6 \text{ m}$

AISI 1006 HR: $S_y = 170 \text{ MPa}$

Cross-section



Column AB/AC

$k = \sqrt{I/A} = \left[\frac{7.336\times10^{-8}}{3.04\times10^{-4}}\right]^{1/2} = 0.009425$

$\left(\frac{l}{k}\right) = \frac{0.433}{0.00943} = 45.92$

Since section is square, lower C is more dangerous → pinned-pinned in plane, $C = 1$

$\left(\frac{l}{k}\right)_{cr} = \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} = \left(\frac{2\pi^2(1)(200\times10^9)}{170\times10^6}\right)^{1/2} = 152.4$

$\left(\frac{l}{k}\right) < \left(\frac{l}{k}\right)_{cr} \rightarrow$ Johnson

$P_{cr} = A\left[S_y - \left(\frac{S_y}{2\pi}\frac{l}{k}\right)^2\frac{1}{CE}\right]$

$= (1.84\times10^{-4})\left[(170\times10^6) - \left(\frac{170\times10^6}{2\pi}(45.92)\right)^2\frac{1}{(1)(200\times10^9)}\right]$

$= \boxed{29860\ N}$

$n = \frac{P_{cr}}{F_{AB}} = \frac{29860}{4298} = \boxed{6.95}$

Column BD/CE

$k = 0.009425$

$\left(\frac{l}{k}\right) = \frac{0.6}{0.00943} = 63.7$

In plane more dangerous → $C = 1$

$\left(\frac{l}{k}\right)_{cr} = \left(\frac{2\pi^2(1)(200\times10^9)}{170\times10^6}\right)^{1/2} = 152.4$

$\left(\frac{l}{k}\right) < \left(\frac{l}{k}\right)_{cr} \rightarrow$ Johnson

$P_{cr} = A\left[S_y - \left(\frac{S_y}{2\pi}\frac{l}{k}\right)^2\frac{1}{CE}\right]$

$= (1.84\times10^{-4})\left[(170\times10^6) - \left(\frac{170\times10^6}{2\pi}(63.7)\right)^2\frac{1}{(1)(200\times10^9)}\right]$

$= \boxed{28547\ N}$

$n = \frac{P_{cr}}{F_{BD}} = \frac{28547}{2149} = \boxed{13.28}$

The smallest factor of safety for members in compression in the frame was $\boxed{n = 6.95}$, suggesting that the frame is very safe against buckling.

Part 3: Fatigue

Assuming 99.9% reliability and operation at room temperature, the feeder side members under the most load (AB and AC) were evaluated for fatigue failure.

PART III: FATIGUE

AISI 1006 HR steel: $S_{ut} = 300$ MPa

$k_a = 38.6(300)^{-0.65} = 0.947$

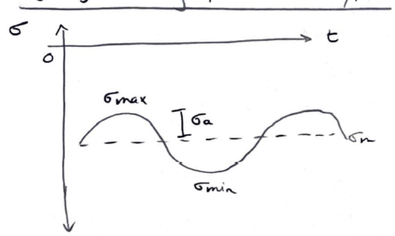
$k_b = 1$, $k_c = 0.85$ (axial)

$k_d = 1$, $k_e = 0.753$ (assume room temp, 99.% reliable)

$S_e = 0.947 \times 1 \times 0.85 \times 1 \times 0.753 \times (0.5 \times 300) = \boxed{90.9\ MPa}$

Assume no stress concentrations

Cycling loading for bar AB/AC



$\sigma_{max} = \frac{F_{min}}{A} = \frac{-3562.3}{1.84\times10^{-4}} = -19.36$ MPa

$\sigma_{min} = \frac{F_{max}}{A} = \frac{-4298}{1.84\times10^{-4}} = -23.36$ MPa

$\sigma_a = \frac{\sigma_{max}-\sigma_{min}}{2} = \frac{-19.36+23.36}{2} = 2$ MPa

$\sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2} = \frac{-19.36-23.36}{2} = -21.36$ MPa

Since $\sigma_m < 0$, we don't use Goodman eqn

Instead, according to Eq. 8-42: $n_f = \frac{S_e}{\sigma_a} = \frac{90.9}{2} = \boxed{45.5}$

Since the mean stress in the member is compressive, the Goodman equation was not used for analysis. Instead, Eq. 8-42 from Shigley was employed. It was determined that the frame would have infinite life with a safety factor of $\boxed{n_f = 45.5}$, suggesting that the frame is very safe against fatigue failure.

Part 4: Power Screw

According to Prof. Hashemi's advice, an experiment was conducted to quantify the force required to push the beans along the drum. This was done by 3D printing mockups of the drum and power screw/push platform assembly. ABS plastic was used as its friction properties are like those of steel on steel. Prof. Hashemi suggested modeling the problem of bean movement as that of a piston experiencing wall shear stress within a fluid. In the worst-case scenario in which the beans form a fully compacted cylinder between the two platforms, such a model may yield a governing equation of the form:

$$F = (\varepsilon d)\pi DL$$

Where d is the bean diameter, D is the drum diameter, and L is the distance between the push platforms. This model suggests that the resistance force is directly proportional to the area multiplied by the grain size and an experimental constant, ε. While we recognize the real system may exhibit non-linearity, it is assumed that the experimental results may be scaled linearly to find the value of F for the real machine.

In the experiment, coffee beans were loaded into the drum and a force gauge was used to measure the force required to begin movement of the beans, thus signaling the transition from static to kinematic friction. The drum was secured to the table to ensure that it would not move when the force gauge was pushed against it. A total of 10 force measurements were obtained, and the average force value was calculated and used in the governing equation above. The experimental setup is shown below:

Values of d, D, L were measured and used to find the proportionality constant α. The results are tabulated below:

Quantity	Value	Units
d	8	mm
D	88	mm
L	50	mm
F	2.3	N
ε	$\dfrac{2.3}{\pi \times 8 \times 88 \times 50} = 2.08 \times 10^{-5}$	N/mm3

The real drum dimensions and spacing between plates were determined by using the density of coffee beans, approximately 750 $\frac{kg}{m^3}$. Assuming that the machine should be capable of sorting 75 kg with a drum ID of 700 mm and power screw of OD 3”, or 76.2 mm:

$$V = \frac{m}{\rho} = \frac{75\ kg}{750\,\frac{kg}{m^3}} = 0.1\ m^3$$

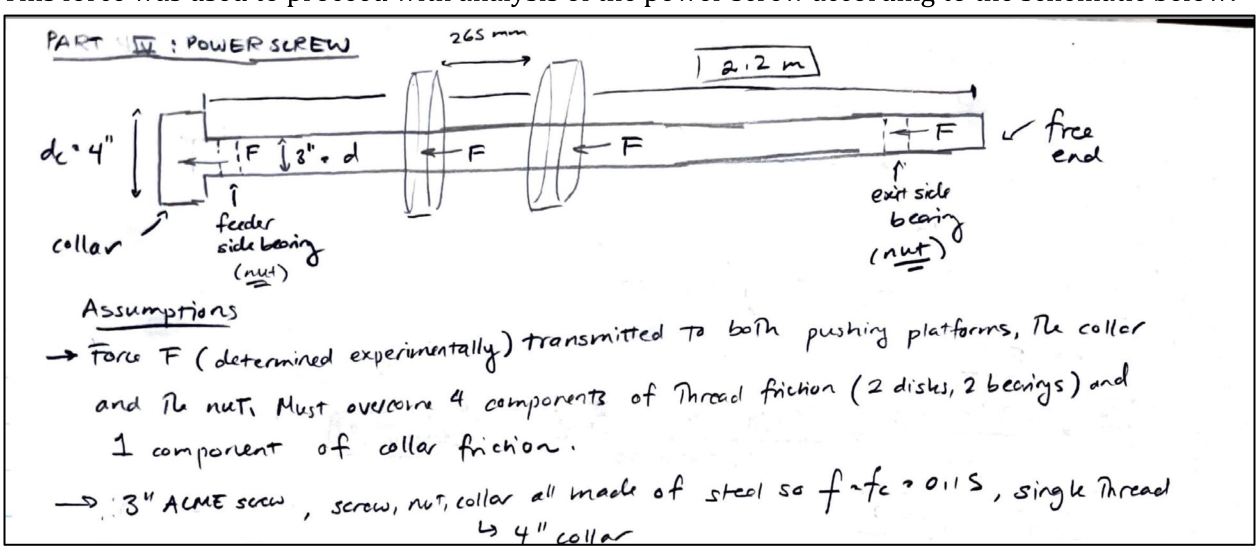
$$A = \frac{\pi}{4}(D^2 - d_s^2) = \frac{\pi}{4}(0.7^2 - 0.0762^2) = 0.38\ m^2$$

$$L = \frac{V}{A} = \frac{0.1}{0.38} = 0.265\ m$$

These values of $D$and L were plugged into the equation to determine the value of F. It was assumed that in the worst-case scenario, the average coffee bean would be slightly larger than those used in the setup, approximately 10 mm in diameter.

$$F = (\varepsilon d)\pi D L = \left(2.08 \times 10^{-5}\,\frac{N}{mm^3}\right)(10\ mm)\pi(700\ mm)(265\ mm) = 121.2\ N = 22.75\ lbf$$

This force was used to proceed with analysis of the power screw according to the schematic below:



It is important to note that for this mechanism to function, the disks must be connected to linear rails on the inside surface of the drum. Without these rails, the disks would simply rotate with the power screw and not cause linear motion of the coffee beans. It was assumed that a collar of $d_c = 4"$ would be used on the feeder side, while the exit side would remain free on the end. Each support bearing would act as a nut, in addition to the interior threads on each push platform/disk. It was also assumed that the force F would be transmitted equally to each threaded section of interest (1 collar, 4 "nuts") as the entire system must be in equilibrium. Therefore, the torque required to start pushing the beans forward must overcome 5 total components of friction: 4 from thread interactions, and 1 from the thrust collar. Using an ACME power screw, the governing equation then becomes:

$$T = 4\left[\frac{Fd_m}{2}\left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - fl\sec\alpha}\right)\right] + \frac{f_c F d_c}{2}$$

The corresponding analysis is shown below:

Required Torque

$F = 121.2\ N = 27.25\ lb$

$$T = 4\left[\frac{Fd_m}{2}\left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - fl\sec\alpha}\right)\right] + \frac{f_c F d_c}{2}$$

Screw parameters: $p = \frac{1}{2}"$, $d_m = d - \frac{p}{2} = 2.75"$; $d_r = d - p = 2.5"$; $l = np = \frac{1}{2}"$, $\alpha = 14.5° \rightarrow \sec\alpha = 1.033$

$$T = 4\left[\frac{(27.25)(2.75)}{2}\left(\frac{(0.5) + \pi(0.15)(2.75)(1.033)}{\pi(2.75) - (0.15)(0.5)(1.033)}\right)\right] + \frac{(0.15)(27.75)(4)}{2} = 32.2 + 8.6 = \boxed{40.8\ lb\text{-}in}$$

Therefore, the power train must be able to deliver at least $\boxed{40.8\ lb \cdot in}$ of torque to the power screw.

The power screw was then analyzed for failure due to thread damage and buckling to select an appropriate material. First, the state of stress at the first thread was determined, assuming the first thread bears 38% of the load:

State of Stress

Assuming first thread takes 38% of the load, such that $\frac{1}{n_t} = 0.38$

$$\sigma_x = \frac{6F}{\pi d_r n_t p} = \frac{6(27.25)(0.38)}{\pi(2.5)(0.5)} = 15.82\ psi \qquad \tau_{xy} = \sigma_z = 0$$

$$\sigma_y = -\frac{4F}{\pi d_r^2} = -\frac{4(27.25)}{\pi(2.5)^2} = -5.55\ psi$$

$$\tau_{xz} = \frac{4T}{\pi d_r^2 n_t p} = \frac{4(40.8)(0.38)}{\pi(2.5)^2(0.5)} = 6.32\ psi$$

$$\tau_{yz} = \frac{16T}{\pi d_r^3} = \frac{16(40.8)}{\pi(2.5)^3} = 13.3\ psi$$

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)\right]^{1/2}$$

$$= \frac{1}{\sqrt{2}}\left[(15.82 + 5.55)^2 + (15.82)^2 + (-5.55)^2 + 6(6.32^2 + 13.3^2)\right]^{1/2} = \boxed{31.93\ psi}$$

It was found that the power screw was under almost no effective stress due to its large size, with $\boxed{\sigma' = 31.93\ psi}$. Therefore, it was assumed as a first guess that AISI 1006 HR steel, the material used for the frame, would be suitable. This material was used to perform the corresponding buckling analysis, assuming that the feeder end is fixed, and the exit end is free:

<u>Buckling</u>, w/ $S_y = 24$ ksi

$$k = \sqrt{I/A} = \frac{d_m}{4} = \frac{2.75}{4} = 0.6875''$$

End conditions: fixed-free $\rightarrow$ $C = \frac{1}{4}$

$$\frac{l}{k} = \frac{2.2\,m \times 39.37\,in/m}{0.6875\,in} = 126$$

$$\left(\frac{l}{k}\right)_{cr} = \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} = \left(\frac{2\pi^2 (1/4)(50\times10^6)}{24000}\right)^{1/2} = 75.54$$

$$\left(\frac{l}{k}\right) > \left(\frac{l}{k}\right)_{cr} \rightarrow \underline{\text{Euler}}$$

$$P_{cr} = \frac{C\pi^2 EI}{l^2} = \frac{C\pi^2 E \frac{\pi}{64} d^4}{l^2} = \frac{(1/4)(\pi^2)(30\times10^6)\frac{\pi}{64}(2.75)^4}{(2.2\times39.37)^2} = \boxed{27700\ lbf}$$

$$n = \frac{P_{cr}}{F} = \frac{27700}{22.75} = \boxed{1217}$$

Using AISI 1006 HR steel, the power screw has a factor of safety of $\boxed{n = 1217}$ against buckling, indicating a very high level of safety.

Summary of Components

In this memo, we have analyzed the frame as a truss under cyclic loading. Analysis was performed to check against static yielding, buckling, and fatigue failure modes. It was determined that using rectangular tubes made AISI 1006 HR steel with a 25 mm × 25 mm cross section with 2 mm thickness would provide a very safe frame with infinite fatigue life. Due to the high factors of safety, inspection and maintenance of the machine should be minimal, which is desirable for the intended use case.

Furthermore, we analyze the torque required to start pushing the coffee beans during the sorting process. The force required to push the beans was determined experimentally and plugged into design equations for power screws. The result will be used to design the motor & gearbox assembly that makes up the machine's power train. It was also determined that the power screw would be very safe against yielding and buckling if made of AISI 1006 HR steel.

4 CONNECTIONS

Introduction to Connections

The design of the coffee bean size-sorting machine contains multiple connection points. The featured methods of connection include joining truss members together with a connecting plate, welding the shaft bearing to the truss frame, and bolting a spacer between the push platforms. Rather than using screws to attach the bearing to the truss frame, welding was implemented for a smoother connection. Forces generated during machine use and requirements for durability create a need for the connections to be reliable and sturdy. The purpose of this memo is to analyze each connection and determine the best screw, bolt, and type of weld for each connection. The CAD model in Figure 1 below shows the draft of the design with dimensions in mm.

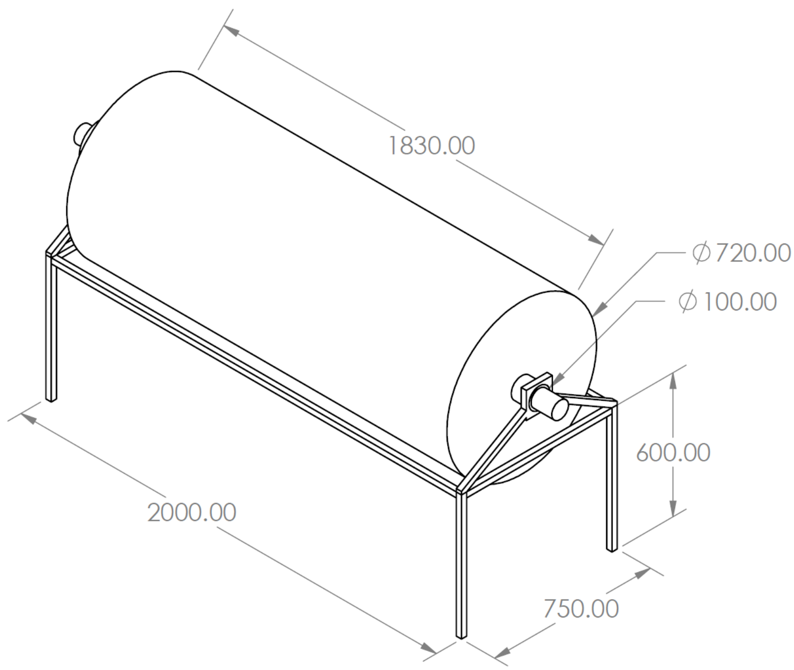


Figure 6: Updated CAD Model Used for Analysis

Loading Conditions & Simplifying Assumptions

The loading conditions used throughout this report are taken from calculations performed in the component design memo and will be referenced as appropriate.

Part 1: Joining Truss Members with Connecting Plate

The first connection of interest is the attachment of the truss members at the joint circled in Figure 2 below:

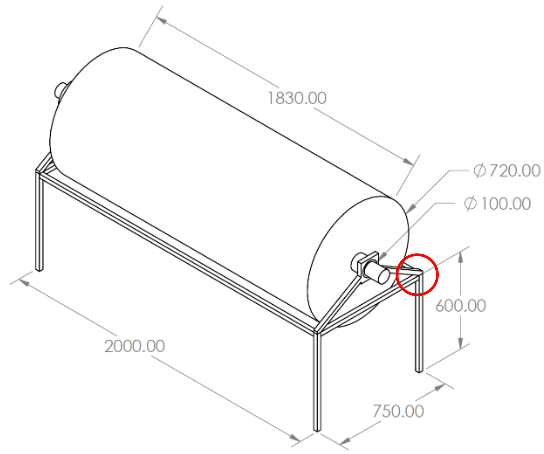


Figure 7: CAD Model with Joint of Interest Highlighted

It was decided to create this connection by attaching the members to a mounting plate using three evenly-spaced bolts about the joint. The member forces found in the component design memo were used to determine the reaction forces exerted on the joint, which in turn subject the bolts to direct and indirect shear loading. The maximum loading conditions from the component design were used to determine the most critical bolt and perform static failure analysis under worst-case conditions. The minimum loading conditions were then used to determine the mean and alternating stresses on the critical bolt for fatigue analysis.

Diagram and Loading Conditions

To design the mounting plate, it was assumed that the bolts would be spaced 50 mm from the joint *O* on each member, as shown below in Figure 3:

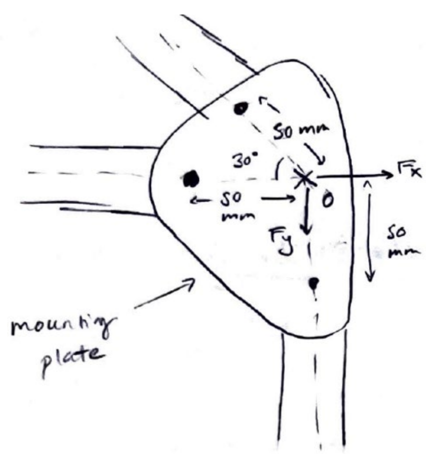


Figure 8: Diagram of Joint with Mounting Plate & Bolts

The forces F_x and F_y acting on the joint were determined from the component analysis. Minimum and maximum values are collected below:

$$(F_x)_{min} = 3085\ N$$

$$(F_x)_{max} = 3722\ N$$

$$(F_y)_{min} = 1781.2\ N$$

$$(F_x)_{max} = 2149\ N$$

Centroid of Bolt Geometry

Since the bolt geometry is not symmetric, the centroid must be calculated as shown below. As a first guess, it will be assumed the bolts are M4 to calculate the areas; however, if all the bolts are the same size, the centroid location will be the same as the areas will cancel. Using point O as the origin:

$$\overline{x} = \frac{\sum x_i A_i}{\sum A_i} = \frac{0+(-50)\left(\frac{\pi}{4}\times 4^2\right)+(-50\cos 30°)\left(\frac{\pi}{4}\times 4^2\right)}{3\times\frac{\pi}{4}\times 4^2} = -31.1\ mm$$

$$\overline{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{0+(50\sin 30°)\left(\frac{\pi}{4}\times 4^2\right)+(-50)\left(\frac{\pi}{4}\times 4^2\right)}{3\times\frac{\pi}{4}\times 4^2} = -8.33\ mm$$

Determining Resultant Shear Forces on Bolts

Based on the calculated centroid, the bolt geometry is shown below in Figure 4:

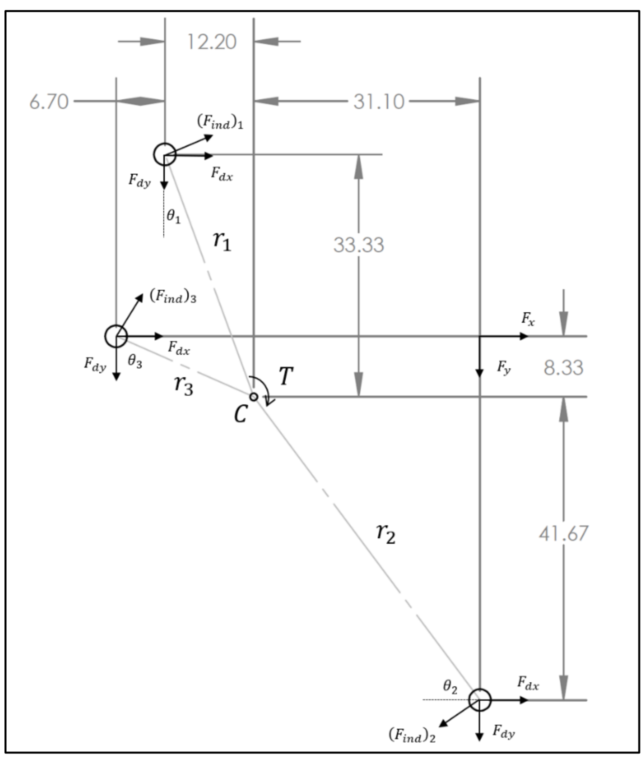


Figure 9: Bolt Geometry and Forces

For worst-case conditions, the maximum values of F_x and F_y were used for analysis. The direct shear force on each bolt was determined by sharing the loads F_x and F_y equally:

$$F_{dx} = \frac{F_x}{3} = \frac{3722}{3} = 1241\ N$$

$$F_{dy} = \frac{2149}{3} = 716.3\ N$$

To calculate the indirect shear force, the torque about the centroid was first calculated:

$$T = (0.00833 \times 3722) + (0.0311 \times 2149) = 97.85\ N \cdot m\ (CW)$$

Next, the radial distance from the centroid to each bolt was determined:

$$r_1 = \sqrt{12.2^2 + 33.33^2} = 35.5\ mm$$

$$r_2 = \sqrt{31.1^2 + 41.67^2} = 52\ mm$$

$$r_3 = \sqrt{18.9^2 + 8.33^2} = 20.66\ mm$$

$$\sum r_i^2 = 35.5^2 + 52^2 + 20.66^2 = 4391\ mm^2 = 4.391 \times 10^{-3}\ m^2$$

This information was then used to calculate the indirect shear on each bolt:

$$(F_{ind})_1 = \frac{Tr_1}{\sum r_i^2} = \frac{97.85 \times 0.0355}{4.391 \times 10^{-3}} = 791\ N$$

$$(F_{ind})_2 = \frac{Tr_2}{\sum r_i^2} = \frac{97.85 \times 0.052}{4.391 \times 10^{-3}} = 1159\ N$$

$$(F_{ind})_3 = \frac{Tr_3}{\sum r_i^2} = \frac{97.85 \times 0.02066}{4.391 \times 10^{-3}} = 461\ N$$

Finally, the resultant force on each bolt was determined by breaking the direct and indirect shears into x and y components and finding the resultant magnitude. For bolt 1:

$$\theta_1 = \tan^{-1}\left(\frac{12.2}{33.33}\right) = 20.1°$$

$$(F_{Rx})_1 = F_{dx} + (F_{ind})_{1,x} = 1241 + 791 \cos 20.1° = 1984\ N$$

$$\left(F_{Ry}\right)_1 = F_{dy} - (F_{ind})_{1,y} = 716.3 - 791 \sin 20.1° = 445\ N$$

$$(F_R)_1 = \sqrt{1984^2 + 445^2} = 2033\ N$$

Repeating the calculations for bolt 2:

$$\theta_2 = \tan^{-1}\left(\frac{41.67}{31.1}\right) = 53.26°$$

$$(F_{Rx})_2 = F_{dx} - (F_{ind})_{2,x} = 1241 - 1159 \sin 53.26° = 312.2\ N$$

$$\left(F_{Ry}\right)_2 = F_{dy} + (F_{ind})_{2,y} = 716.3 + 1159 \cos 53.26° = 1410\ N$$

$$(F_R)_2 = \sqrt{312.2^2 + 1410^2} = 1444\ N$$

Repeating for bolt 3:

$$\theta_3 = \tan^{-1}\left(\frac{18.9}{8.33}\right) = 66.2°$$

$$(F_{Rx})_3 = F_{dx} + (F_{ind})_{3,x} = 1241 + 461 \cos 66.2° = 1427\ N$$

$$\left(F_{Ry}\right)_3 = F_{dy} - (F_{ind})_{3,y} = 716.3 - 461 \sin 66.2° = 295\ N$$

$$(F_R)_2 = \sqrt{1427^2 + 295^2} = 1457\ N$$

These calculations show that **bolt 1 is most dangerous, with** $\boxed{\boldsymbol{F_{R,max} = 2033\ N}}$. This bolt will be used in subsequent analysis.

Required Bolt Length

Before calculating the shear stress on the bolt, it is important to know the required bolt length to see whether the shear plane acts on the threads or the shank. It is assumed that the mounting plate is 5 mm thick, and from the component design, it is known that the hollow rectangular tubing of the frame is 2 mm thick. It is also assumed that nuts will be used rather than tapped holes due to the thin material. Starting analysis with M4x0.7 bolts, Table 8-7 may be used to determine the required bolt length:

$$L > l + H$$

The grip length is $l = 5 + 2 = 7\ mm$. Table A-31 does not provide recommended nut heights for M4 bolts. However, online resources suggest $H = 3.2$ mm. Therefore, $L > 7 + 3.2 = 10.22\ mm$. The threaded length is given by:

$$L_T = 2d + 6mm = 2(4) + 6 = 14\ mm$$

Since L_T is greater than the minimum length L, we round up to $L = 14$ mm such that the entire bolt is threaded (i.e. $l_d = 0$). Since there is no shank, the shear plane acts on the threads, and the minor diameter area must be used.

Static Failure Analysis

From Table 8-1 for M4 bolts, $A_r = 7.75\ mm^2$. The maximum shear stress experienced by bolt 1 is then given by:

$$\tau = \frac{F_{R1}}{A_r} = \frac{2033}{7.75} = 262.3\ MPa$$

For first calculations, Class 8.8 bolts are checked, as they represent the average grade. From Table 8-11, $S_y = 660\ MPa$. The factor of safety against static failure is the given by DET theory as:

$$n = \frac{S_y}{\sqrt{3}\tau} = \frac{660}{\sqrt{3} \times 262.3} = \boxed{1.45}$$

While marginally safe, this factor of safety was deemed unsatisfactory for the intended application since the goal is to minimize maintenance/inspection for this machine. Thus, calculations were **repeated with M6x1, Class 9.8 bolts**. The increased size and quality will make the bolts more expensive but should lower costs associated with failure over the life of the product.

From Table A-31 for M6 bolts, $H = 5.2\ mm$. Therefore, the new minimum length is: $L > 7 + 5.2 = 12.2\ mm$. The threaded length is $L_T = 2d + 6 = 2(6) + 8 = 18\ mm$. Since once again the threaded length exceeds the minimum suggested length, we round the bolt length to 18 mm and use the minor diameter area in calculations. From Table 8-1, $A_r = 17.9\ mm^2$, and from Table 8-11, $S_y = 720\ MPa$. Therefore:

$$\tau_{max} = \frac{F_{R1}}{A_r} = \frac{2033}{17.9} = 113.6\ MPa$$

$$n = \frac{S_y}{\sqrt{3}\tau_{max}} = \frac{720}{\sqrt{3} \times 113.6} = \boxed{3.66}$$

This safety factor was deemed satisfactory. **Thus, the suggested fasteners for this joint based on static failure are three M6x1, 18 mm long, fully threaded bolts.**

Fatigue Failure Analysis

Fatigue was also considered, since from the component analysis it was found that the member forces are fluctuating. First, the direct, indirect, and resultant shear forces on bolt 1 under the minimum loading conditions were calculated:

$$F_{dx} = \frac{F_x}{3} = \frac{3085}{3} = 1028\ N$$

$$F_{dy} = \frac{1781.2}{3} = 594\ N$$

$$T = (0.00833 \times 3085) + (0.0311 \times 1781.2) = 81\ N \cdot m\ (CW)$$

$$(F_{ind})_1 = \frac{Tr_1}{\sum r_i^2} = \frac{81 \times 0.0355}{4.391 \times 10^{-3}} = 655\ N$$

$$(F_{Rx})_1 = F_{dx} + (F_{ind})_{1,x} = 1028 + 655 \cos 20.1° = 1643\ N$$

$$\left(F_{Ry}\right)_1 = F_{dy} - (F_{ind})_{1,y} = 594 - 655\sin 20.1° = 369\ N$$

$$F_{R,min} = \sqrt{1643^2 + 369^2} = 1684\ N$$

The minimum shear stress is then given by:

$$\tau_{min} = \frac{F_{R,min}}{A_r} = \frac{1684}{17.9} = 94\ MPa$$

The alternating and mean stresses are then given by:

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{113.6 - 94}{2} = 9.8\ MPa$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{113.6 + 94}{2} = 103.8\ MPa$$

Finally, the Goodman equation was used to determine the safety factor for infinite fatigue life. For a Class 9.8 bolt, $S_{ut} = 900\ MPa$ (Table 8-11), and $S_e = 140\ MPa$ (Table 8-17). Therefore:

$$n_f = \left(\frac{\sqrt{3}\tau_a}{S_e} + \frac{\sqrt{3}\tau_m}{S_{ut}}\right)^{-1} = \left(\frac{\sqrt{3}\times 9.8}{140} + \frac{\sqrt{3}\times 103.8}{900}\right)^{-1} = \boxed{3.11}$$

With a reasonably high safety factor, the joint should not fail due to fatigue.

Part 2: Welding Shaft Bearing to Truss Frame

The second connection of interest is the attachment of the truss members to the bearing at the joint circled in Figure 5 below:

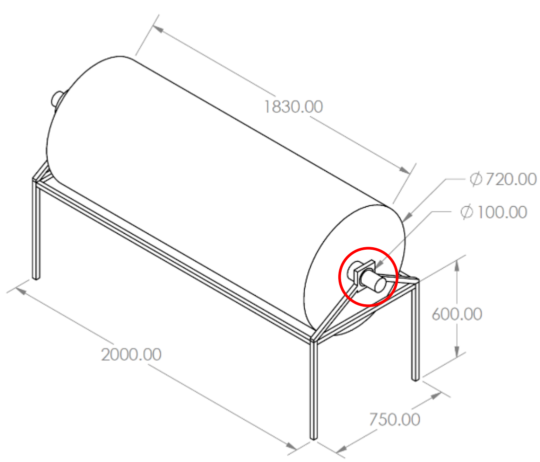


Figure 5: CAD Model with Joint of Interest Highlighted

It was decided to create this connection by welding the members to the bearing using E60xx. T, creating one cohesive part. The minimum and maximum forces that act on the bearing were found in the component design memo and were used to determine the reaction forces exerted on the welds, which in turn subject the welds to direct shear and indirect bending loading. The maximum loading conditions from the component design were used to determine the most critical weld location and perform static failure analysis under worst-case conditions. The minimum loading conditions were then used to determine the mean and alternating stresses on the critical weld location for fatigue analysis.

Diagram and Loading Conditions

It was assumed that the welded bearing acts as a beam of length 125 mm with a fluctuating center load. The two members are joined at an angle of 30° and are made of AISI 1006 HR steel. These welded members act as fixed supports on both sides, as shown below in Figure 6:

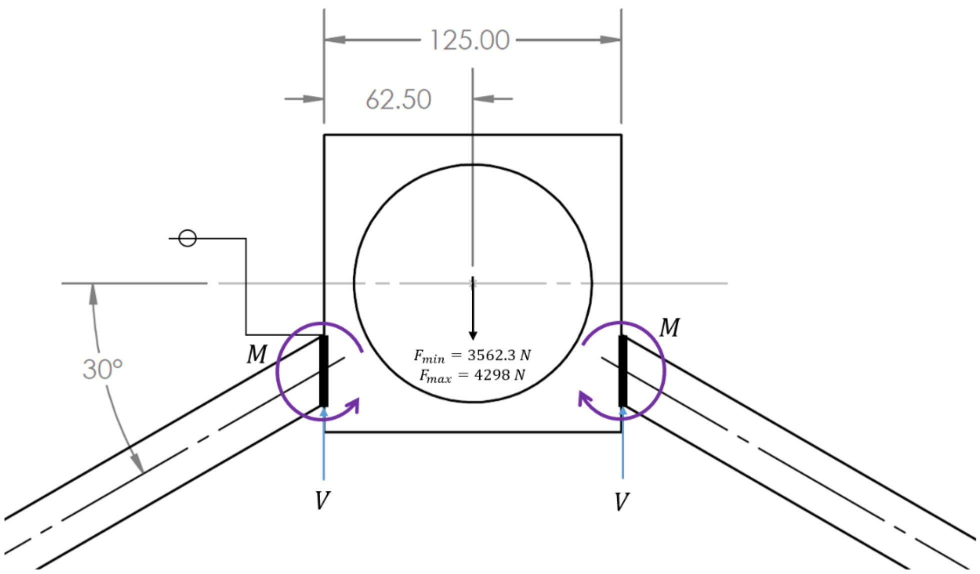


Figure 6: Diagram of Welded Bearing

The forces F_{min} and F_{max} acting on the beam were determined from the component analysis. Minimum and maximum values are collected below:

$$F_{min} = 3562.3\ N$$

$$F_{max} = 4298\ N$$

Using Table A-9 for fixed supports with a center load, the minimum and maximum reaction moments on each side of the bearing were calculated:

$$M_{min} = \frac{F_{min} l}{8} = \frac{(3562.3)(125)}{8} = 55660.94\ N \cdot mm$$

$$M_{max} = \frac{F_{max} l}{8} = \frac{(4298)(125)}{8} = 67156.25\ N \cdot mm$$

Centroid of Weld Geometry

Since the weld geometry is symmetric, we can use Table 9-2 to determine the centroid and unit second moment of area, I_u, as shown below:

$$\bar{x} = \frac{b}{2} = \frac{25}{2} = 12.5\ mm$$

$$\bar{y} = \frac{d}{2} = \frac{25}{2} = 12.5\ mm$$

$$I_u = \frac{d^2}{6}(3b + d) = \frac{(25)^2}{6}(3(25) + 25) = 10416.67\ mm^3$$

Static Failure Analysis

The 3D loading configuration is shown below in Figure 7. E60xx is used for the welds with a throat height of h = 6 mm. The members are hollow square tubes with a cross-sectional side length of 25 mm and are made of AISI 1006 HR steel.

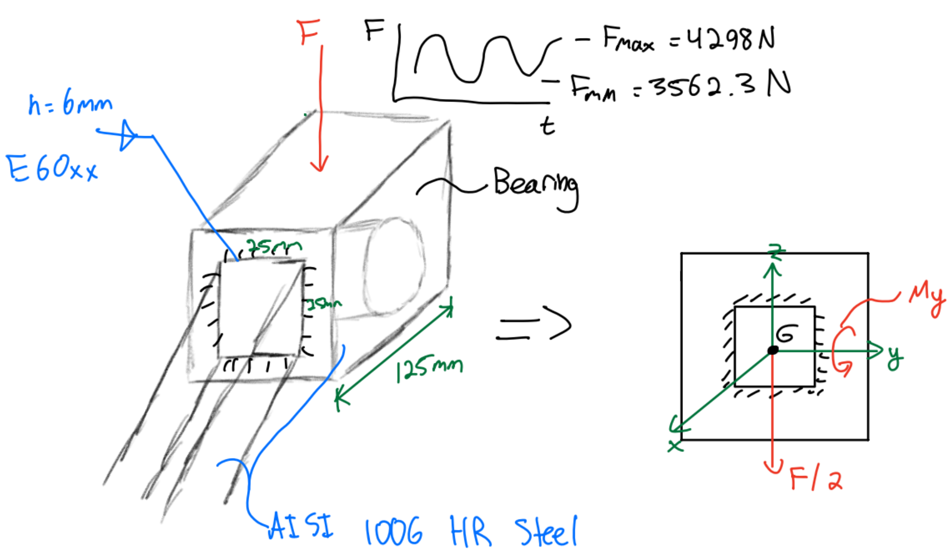


Figure 7: 3D Diagram of Welded Bearing with Loads

Each member shares half the applied load and is subject to a bending moment as calculated above. To analyze the safety factor against static yielding, the values of F_{max} and M_{max} are used.

The direct shear force on the throat is calculated by:

$$\tau_d = \frac{F_{max}/2}{0.707hL_{weld}} = \frac{4298/2}{0.707(6)(100)} = 5.07\ MPa$$

The indirect shear force on the throat due to bending is calculated by:

$$\tau_{ind} = \frac{M_{max}c}{0.707hI_u} = \frac{(67156.25)(12.5)}{0.707(6)(10416.67)} = 19.0\ MPa$$

Due to the symmetric nature of the design, the resultant shear forces will be the same at both the top and bottom welds. However, the top weld is under tension and will therefore be the dangerous location. The result shear force is calculated by:

$$\tau_R = \sqrt{(\tau_d)^2 + (\tau_{ind})^2} = \sqrt{(5.07)^2 + (19.0)^2} = 19.7\ MPa$$

The allowable shear stress for the E60xx weld is given in Table 9-6 as $\tau_{allow} = 18\ ksi$. A conversion factor of 6.895 MPa/ksi is used to convert this allowable shear stress to MPa. The safety factor for the weld is therefore given as:

$$n = \frac{\tau_{allow}}{\tau_R} = \frac{(18)(6.895)}{19.7} = \boxed{6.3}$$

For the bearing base material of AISI 1006 HR steel, the yield strength is given in Table A-20 as $S_y = 170\ MPa$. The safety factor for the base material is therefore given as:

$$n = \frac{0.4S_y}{0.707\tau_R} = \frac{(0.4)(170)}{(0.707)(19.7)} = \boxed{4.88}$$

These safety factors were considered sufficient for static loading, so analysis continued using these weld parameters.

Fatigue Failure Analysis

Fatigue was also considered, since from the component analysis it was found that the center load is fluctuating. The maximum shear stress experienced by the weld is $\tau_{max} = 19.7\ MPa$. Since the center load applied is proportional to the shear stress experienced, the minimum shear stress experienced by the weld is given as:

$$\tau_{min} = \tau_{max}\left(\frac{F_{min}}{F_{max}}\right) = (19.7)\left(\frac{3562.3}{4298}\right) = 16.3\ MPa$$

The shear stress amplitude is given as:

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{19.7 - 16.3}{2} = 1.7\ MPa$$

The mean shear stress is given as:

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{19.7 + 16.3}{2} = 18\ MPa$$

For weld E60xx, the tensile strength is given in Table 9-3 as $S_{ut} = 427\ MPa$. Using an as-forged surface finish, Table 6-2 can be used to calculate k_a:

$$k_a = aS_{ut}^b = (54.9)(427)^{-.758} = 0.5568$$

This value can then be used to calculate the actual endurance strength S_e:

$$S_e = k_a(0.5S_{ut}) = (0.5568)(0.5)(427) = 118.9\ MPa$$

The fatigue stress-concentration for the end of a parallel fillet weld is given in Table 9-5 as $K_{fs} = 2.7$. The Goodman equation is then used to determine the safety factor for infinite fatigue life:

$$n_f = \left(\frac{K_{fs}\sqrt{3}\tau_a}{S_e} + \frac{K_{fs}\sqrt{3}\tau_m}{S_{ut}}\right)^{-1} = \left(\frac{2.7(\sqrt{3})(1.7)}{118.9} + \frac{2.7(\sqrt{3})(18)}{427}\right)^{-1} = \boxed{3.79}$$

The safety factors for the weld, base, and against fatigue failure were all deemed satisfactory. **Thus, the suggested weld for this design is E60xx with a throat height of 6 mm.**

Part 3: Bolting Spacer for Push Platforms

The third and final connection of interest is the push disk connector, which encases the power screw to protect against coffee beans and other sources of dirt. A sketch of the current design is found in Figure 8 below.

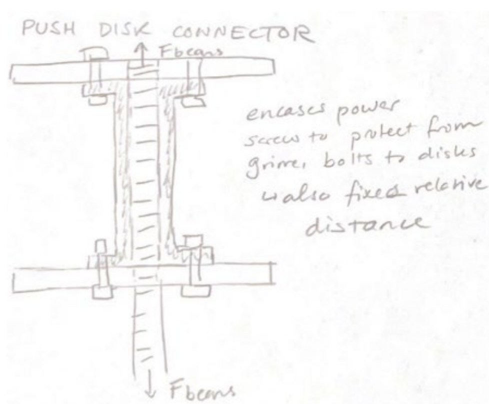


Figure 8: Diagram of Push Disk Connector

As shown in the figure above, four bolts will be used to secure the push disk connector and ensure a fixed relative distance as the power screw rotates to push the disks forward. The force of the coffee beans, determined from the component design report, acts on each side of the push disk connector and results in an axial force applied to each bolt. For calculations, we assume that the spacer is made of plastic, as It does not need to bear a significant load and should be light and cheap.

Diagram and Loading Conditions

Standard M5 bolts will be used for this design. An updated diagram with the necessary dimensions and loading conditions is found in Figure 9 below:

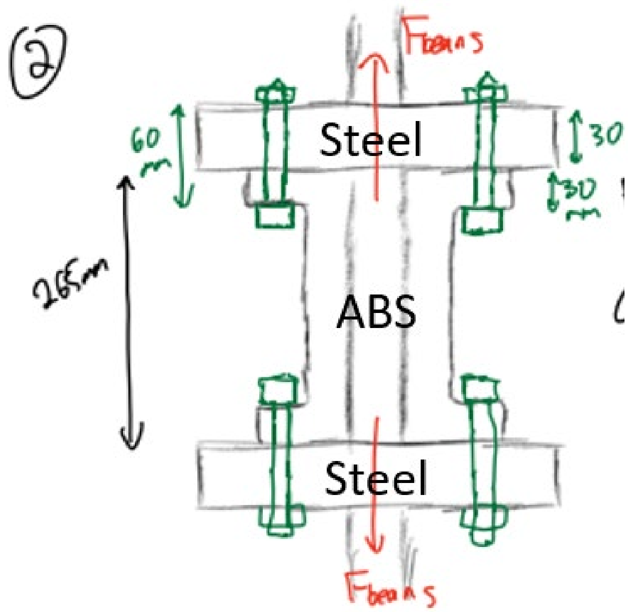


Figure 9: Diagram and Loading of Push Disk Connector

The force F_{beans} was determined from the component analysis and is given as:

$$F_{beans} = 121.2\ N$$

Centroid of Bolt Geometry

Upon evaluating one side of the push disk connector, one can see that the centroid of the two bolts lies directly between them. The axial force is applied at this location, which means that each bolt will only experience a direct force (P) and no indirect force.

Determining Resultant Forces on Bolts

The direct axial force exerted on each bolt is given by:

$$P = \frac{F_{beans}}{2} = \frac{121.2}{2} = 60.6\ N$$

Required Bolt Length and Stiffness

Before calculating the stress on each bolt, it is important to know the required bolt length. Starting analysis with M5 bolts and nuts, Table 8-7 is used to determine the required bolt length:

$$L > l + H$$

As shown in the diagram, the grip length is determined from the 30 mm steel section and the 30 mm ABS section: $l = 30 + 30 = 60\ mm$. From Table A-31, the recommended regular hexagonal nut height for M5 bolts is given as $H = 4.7$ mm. Therefore, $L > 60 + 4.7 = 64.7\ mm$. This value will be rounded up to $L = 65\ mm$. Since $L \leq 125\ mm$ and $d \leq 48\ mm$, the threaded length is given by:

$$L_T = 2d + 6\ mm = 2(5) + 6 = 16\ mm$$

The length of the unthreaded portion in the grip is given by:

$$l_d = L - L_T = 65 - 16 = 49\ mm$$

The length of the threaded portion in the grip is given by:

$$l_t = l - l_d = 60 - 49 = 11\ mm$$

The area of the shank of the M5 bolt is determined using the major diameter:

$$A_d = \frac{\pi}{4} d^2 = \frac{\pi}{4}(5)^2 = 19.63\ mm^2$$

The tensile stress area of the threaded portion of the M5 bolt is given in Table 8-1:

$$A_t = 14.2\ mm^2$$

Assuming the bolts are made of steel with $E = 210\ GPa$, the stiffness of each bolt is calculated by:

$$K_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{(19.63)(14.2)(210000)}{(19.63)(11) + (14.2)(49)} = 64216.3\ N/mm$$

Member Stiffness

As mentioned earlier, the two members of this design are a 30 mm steel section and 30 mm ABS section. The two frustums created to represent this design are shown in Figure 10 below:

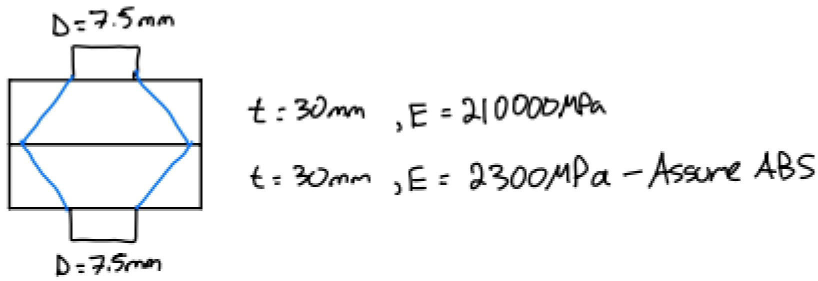


Figure 10: Frustums for Member Stiffness

Each frustum has a thickness $t = 30\ mm$ and a diameter given by:

$$D = 1.5d = 1.5(5) = 7.5\ mm$$

The top frustum is made of steel with $E = 210{,}000\ MPa$, and the bottom frustum is made of ABS with $E = 2{,}300\ MPa$. The individual member stiffnesses are calculated by:

$$K_{m_i} = \frac{0.5774\pi dE}{\ln\left(\dfrac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}\right)}$$

For convenience, this formula was provided in an Excel sheet. The resulting member stiffnesses for the steel (1) and ABS (2) sections are found to be:

$$K_{m_1} = 1388471\ N/mm$$

$$K_{m_2} = 15207\ N/mm$$

The overall member stiffness is then calculated by:

$$K_m = \left(\frac{1}{K_{m_1}} + \frac{1}{K_{m_2}}\right)^{-1} = \left(\frac{1}{1388471} + \frac{1}{15207}\right)^{-1} = 15042\ N/mm$$

Static Failure Analysis

With both bolt stiffness and member stiffness calculated, the value of C is found by:

$$C = \frac{K_b}{K_b + K_m} = \frac{64216.3}{64216.3 + 15042} = 0.81$$

For first calculations, the M5 bolts used will be Metric Grade 4.6, so Table 8-11 is used to obtain the following properties:

$$S_P = 225\ MPa$$

$$S_{ut} = 400\ MPa$$

$$S_y = 240\ MPa$$

These bolts should be serviceable for inspection, so the pretention is calculated by:

$$F_i = 0.75 A_t S_P = (0.75)(14.2)(225) = 2396.25\ N$$

These bolts should also be lubricated, so the torque factor is given in Table 8-15 as $K = 0.18$. Therefore, the torque required to achieve the desired pretention is given by:

$$T = KdF_i = (0.18)(5)(2396.25) = 2156.6\ N \cdot mm$$

The safety factor against **static failure** under the load P is given by:

$$n_L = \frac{A_t S_P - F_i}{CP} = \frac{(14.2)(225) - (2396.25)}{(0.81)(60.6)} = \boxed{16.3}$$

The safety factor against **loosening** under the load P is given by:

$$n_o = \frac{F_i}{(1 - C)P} = \frac{2396.25}{(1 - 0.81)(60.6)} = \boxed{208.1}$$

With relatively high safety factors, the size and grade were deemed satisfactory for static loading, and fatigue was subsequently investigated.

Fatigue Failure Analysis

Before calculating the safety factor against eventual fatigue failure, the endurance strength S_e must be calculated along with the amplitude and mean applied load P. Assuming the threads are cut, the fatigue stress-concentration factor for threaded Metric Grade 4.6 bolts is given in Table 8-16 as $K_f = 2.8$. Therefore, the endurance strength is calculated by:

$$S_e = \frac{1}{K_f}(0.5 S_{ut}) = \frac{1}{2.8}(0.5)(400) = 71.4\ MPa$$

When all the beans are in the drum, the maximum applied load is given by $P_{max} = 60.6\ N$. When all the beans have been sorted, the minimum applied load is given by $P_{min} = 0\ N$.

The applied load amplitude is given as:

$$P_a = \frac{P_{max} - P_{min}}{2} = \frac{60.6 - 0}{2} = 30.3\ MPa$$

The mean applied load is given as:

$$P_m = \frac{P_{max} + P_{min}}{2} = \frac{60.6 + 0}{2} = 30.3\ MPa$$

The safety factor against **fatigue failure** is then calculated by:

$$n_f = \frac{S_{ut}A_t - F_i}{C\left(P_a \frac{S_{ut}}{S_e} + P_m\right)} = \frac{(400)(14.2) - 2396.25}{(0.81)\left(30.3\left(\frac{400}{71.4}\right) + 30.3\right)} = \boxed{20.3}$$

The safety factors against static failure, loosening, and fatigue failure were all deemed satisfactory. **Thus, the suggested fasteners for this design are M5x0.8 Metric Grade 4.6, 65 mm long bolts.** Additionally, the spacer may be made of ABS. Due to the high value of C, most of the load is taken up by the bolts rather than the members. Since the bolts are very safe, we may also assume that the spacer is safe.

Summary of Connections

In this memo, the points of connection for the truss frame, bearing, and spacer were all analyzed, and appropriate design choices were made for each. Calculations involved bolts under shear, bolts under tension, and welds under combined shear and bending. Analysis of each connection was performed to check for both static and fatigue failure. To join the truss members, failure analysis of the most dangerous bolt led to a recommendation of using M6x1, 18mm long, fully threaded bolts. Furthermore, our calculations determined that welding the bearing to the frame using E60xx fillet welds with a throat height of 6mm provides safety against throat failure under static and cyclic conditions. The weld also provides enough support to prevent separation from the base material. Tensile stress analysis was used on the bolts connecting the push disks to the plastic spacer. It was determined that low-grade, small bolts could be used for this connection, though they need to be long to accommodate the grip length. We suggest M5x0.8, 65 mm long bolts for this application.

5 POWER TRAIN

Introduction to Power Train

The design of the coffee bean size-sorting machine features a power train to run the power screw. The development of the power train included picking the motor and designing the gearbox. The best possible motor would be decided based on required speeds that are obtainable with the lead screw selected previously. Forces generated during active motor use and requirements for durability create a need for the gears to be reliable and uphold an acceptable factor of safety. The purpose of this memo is to analyze each part of the gearbox and determine the required power and torque of the motor and the best gear material and specification to handle the calculated power and torque. The CAD model in Figure 1 below shows the draft of the design with dimensions in mm.

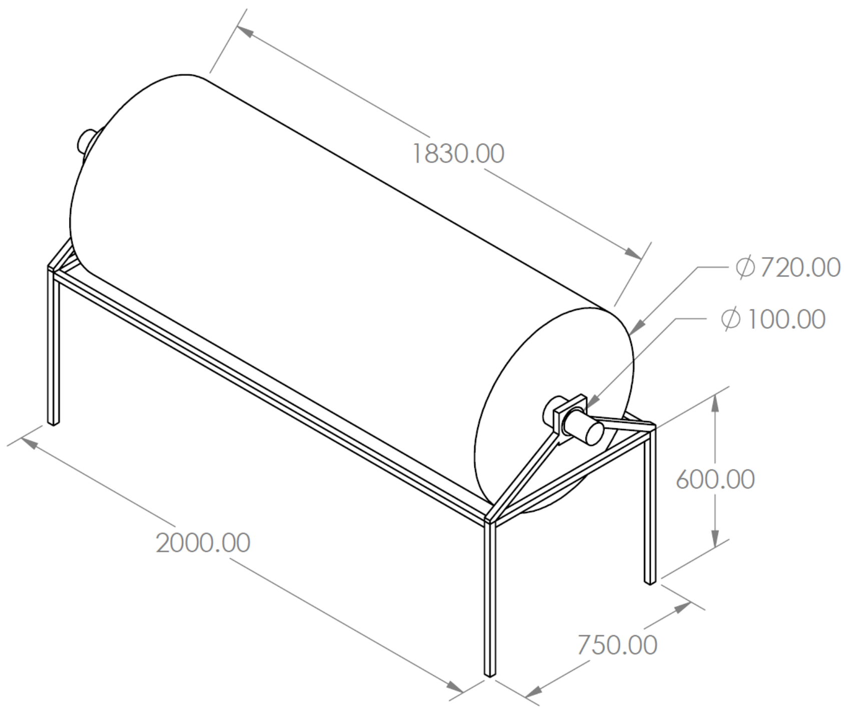


Figure 10: Updated CAD Model Used for Analysis

Loading Conditions & Simplifying Assumptions

The loading conditions used throughout this report are taken from calculations performed in the component design and connections memos and will be referenced as appropriate.

Part 1: Motor Selection

The design of the power train began with specifying the desired speed at which the beans may be sorted. It was decided to **design for a pushing speed of 1 in/s, or 60 in/min.** This would allow each 75 kg batch of coffee beans to be sorted in approximately 3 minutes (including the return motion). With a maximum annual harvest of 500 kg, a micro-mill could sort their entire inventory within 20-30 minutes using this machine.

From the component design, it was found that the required torque to sort the beans is 40.8 lb-in. Additionally, the lead of the single-threaded, 3" ACME power screw used in the design is 0.5" in/rev. This information may be used to determine the power that must be supplied by the motor:

$$n = \frac{v}{l} = \frac{60\ in/min}{0.5\ in/rev} = 120\ rpm$$

$$hp = \frac{Tn}{63000} = \frac{40.8 \times 120}{63000} = 0.078\ hp$$

Looking at the McMaster catalog, the most commonly available motor power that exceeds our requirement is 1/8 hp. For this design, **motor 59825K49, which can deliver 1/8 hp at 475 rpm, is selected.** It is necessary to design a gear train to reduce the speed of rotation to achieve the desired linear speed and meet the minimum torque requirement. By inspection, this gear train needs an overall gear ratio of $e \cong 4$.

Part 2: Gearbox Design

Gear Selection

A two-stage gear reducer is employed for this design, using two identical pairs of gears such that the input and output shafts are collinear. From above, we estimate that an overall gear ratio of 4 is required, so a pinion/gear set with a gear ratio of 2 is selected from the McMaster catalog. For the first iteration, we select the pinion 5172T21 and gear 5172T23 with the following characteristics:

$$P = 16 = \frac{N}{d}$$

$$N_p = 16 \rightarrow d_p = 1"$$

$$N_g = 32 \rightarrow d_g = 2"$$

$$b = 0.75"$$

$$\phi = 20°$$

Finding Output Speed/Torque

A schematic of the gearbox is shown below in Figure 2:

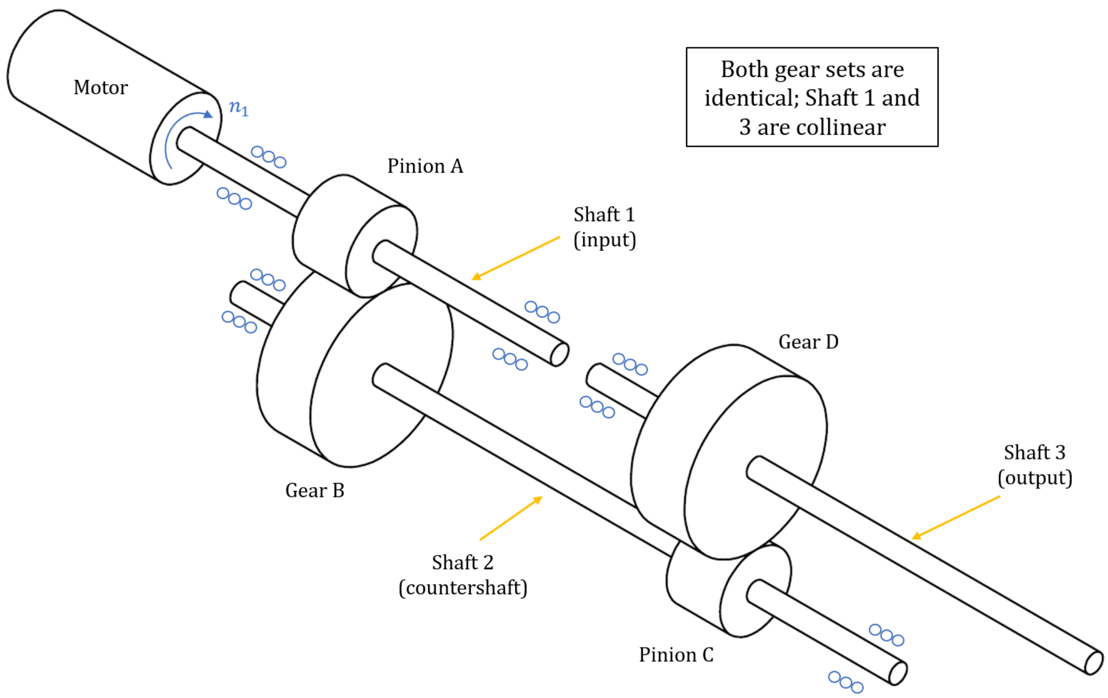


Figure 11: Schematic of Gearbox

The output shaft will be coupled to the power screw, enabling the sorting motion. Gear ratios are then applied to each shaft to determine the corresponding speed and torque. At the input (shaft 1):

$$n_1 = 475 \; rpm$$

$$T_1 = \frac{63000 \; hp}{n_1} = \frac{63000 \times 0.125}{475} = 16.6 \; lb \cdot in$$

For the countershaft (shaft 2):

$$n_2 = n_1\left(-\frac{N_p}{N_g}\right) = 475\left(-\frac{1}{2}\right) = -237.5 \; rpm$$

$$T_2 = T_1\left(\frac{N_g}{N_p}\right) = 16.6(2) = 33.2 \; lb \cdot in$$

At the output (shaft 3):

$$n_3 = n_2\left(-\frac{N_p}{N_g}\right) = -237.5\left(-\frac{1}{2}\right) = \boxed{118.75 \; rpm \cong 120 \; rpm}$$

$$T_2 = T_1\left(\frac{N_g}{N_p}\right) = 33.2(2) = \boxed{66.4\ lb \cdot in > 40.8\ lb \cdot in}$$

This gearbox can output a torque of 66.4 lb-in at a speed of 118.75 rpm, which significantly exceeds the minimum torque requirement and very closely matches the target speed. Thus, the performance of the design is deemed satisfactory and analysis proceeds to determine if the chosen gears can withstand the applied stresses.

Force Analysis

Free body-diagrams of all the gears are shown below in Figure 3:

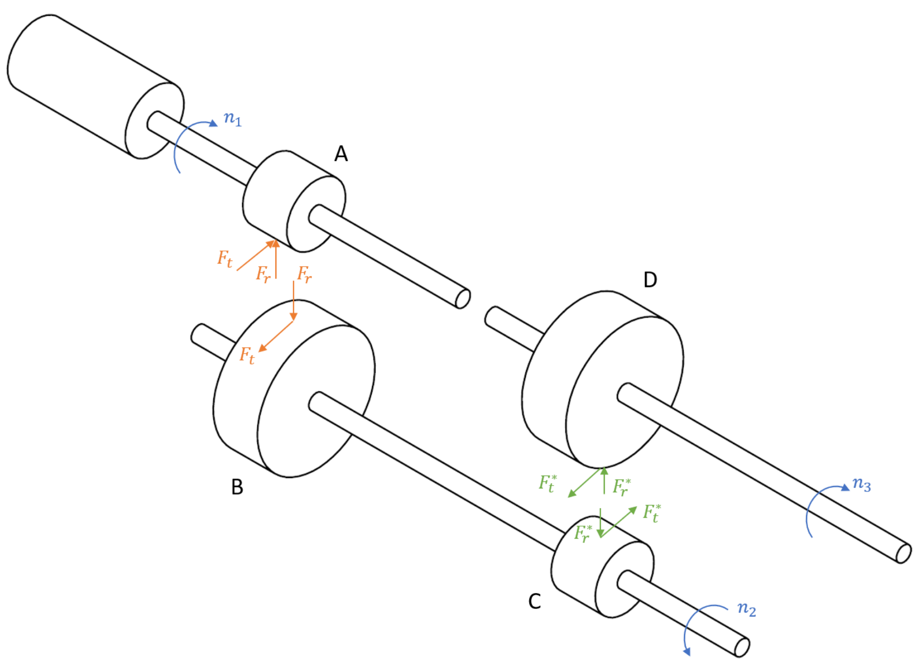


Figure 12: Gear Free-Body Diagrams

Each pinion/gear pair (A/B and C/D) experience equal and opposite contact forces. For pair A/B:

$$F_t = \frac{T_1}{d_p/2} = \frac{16.6}{0.5} = 33.2\ lb$$

$$F_r = F_t \tan 20° = 12.08\ lb$$

For pair C/D:

$$F_t^* = \frac{T_2}{d_p/2} = \frac{33.2}{0.5} = 66.4\ lb$$

$$F_r^* = F_t^* \tan 20° = 24.16\ lb$$

Pinion C, being smaller and subject to the most force, is the most vulnerable gear in this design. Therefore, it will be used for subsequent calculations. Once a safe design is determined for Pinion C, factors of safety for the other gears will also be found to verify the decision.

Stress Analysis

The AGMA equation for bending stress on the gear teeth will be used to analyze safety:

$$\sigma = F_t^* K_o K_v \frac{P}{b} \frac{K_s K_m K_b}{J}$$

To apply the equation, all modifying factors must first be determined. Assuming medium shock at the input and output, $K_o = 1.75$. Based on average quality gears ($Q_v = 7$) and a pitch line velocity of $V = \frac{\pi d_p n_2}{12} = 62$ ft/min, $K_v \cong 1.1$. Based on average mounting, $K_m = 1.6$. Based on $N_p = 16$, $J = 0.275$. Finally, assuming $K_s = K_b = 1$:

$$\sigma = 66.4 \times 1.75 \times 1.1 \times \frac{16}{0.75} \times \frac{1 \times 1.6 \times 1}{0.275} = \boxed{15865\ psi}$$

Next, the allowable stress must be determined:

$$\sigma_{all} = \frac{S_t Y_N}{K_T K_R}$$

For the first iteration, it is assumed that the gears are made of 180 BHN steel with $S_t = 29\ ksi$. Based on infinite life ($Y_N = 1$), room temperature ($K_T = 1$) and 99% reliability ($K_R = 1$):

$$\sigma_{all} = \frac{29000 \times 1}{1 \times 1} = 29000\ psi$$

Finally, the factor of safety for Pinion C may be determined:

$$n_C = \frac{\sigma_{all}}{\sigma} = \frac{29000}{15865} = \boxed{1.83}$$

This design is safe, but only marginally. To increase the factor of safety, increasing the hardness of the steel was first investigated. For 300 BHN, $\sigma_{all} = S_t = 41500$ psi, giving a new safety factor of:

$$n_C = \frac{\sigma_{all}}{\sigma} = \frac{41500}{15865} = \boxed{2.62}$$

While making the gears out of harder material increases safety, the gain is limited in comparison to the cost of using stronger materials. Rather than continuing to test more premium materials, the design is modified to use larger gears with the same gear ratio.

Selecting New Gears to Increase Safety

For the second iteration, pinion 5172T52 and gear 5172T54 will be used:

$$P = 10 = \frac{N}{d}$$

$$N_p = 20 \rightarrow d_p = 2"$$

$$N_g = 40 \rightarrow d_g = 4"$$

$$b = 1.25"$$

$$\phi = 20°$$

The forces applied to the new gears are:

$$F_t = \frac{T_1}{d_p/2} = \frac{16.6}{1} = 16.6\ lb$$

$$F_r = F_t \tan 20° = 6.04\ lb$$

$$F_t^* = \frac{T_2}{d_p/2} = \frac{33.2}{1} = 33.2\ lb$$

$$F_r^* = F_t^* \tan 20° = 12.06\ lb$$

Stress analysis on Pinion C requires recalculation of K_v and J. With a pitch line velocity of $\frac{\pi d_p n_2}{12} =$ 124 ft/min, $K_v \cong 1.15$. With $N_p = 20$ and $N_g = 40, J \cong 0.35$. The new bending stress is then:

$$\sigma = 33.2 \times 1.75 \times 1.1 \times \frac{10}{1.25} \times \frac{1 \times 1.6 \times 1}{0.35} = \boxed{2443.5\ psi}$$

Using these gears reduces the bending stress by more than 6 times. Using 180 BHN steel again:

$$n_C = \frac{29000}{2443.5} = \boxed{11.9}$$

The safety factor is now very large, so cheaper materials are investigated. For AGMA Cast Iron Grade 40, $\sigma_{all} = S_t = 13000\ psi$. Using this material gives:

$$n_C = \frac{13000}{2443.5} = \boxed{5.32}$$

This design provides a good balance between safety and cost. For completeness, safety factors for the other gears are computed as well:

$$\sigma_A = 1248.2\ psi \rightarrow n_A = \frac{13000}{1248.2} = \boxed{10.41}$$

$$\sigma_B = 1092.3\ psi \rightarrow n_A = \frac{13000}{1092.3} = \boxed{11.9}$$

$$\sigma_D = 2185\ psi \rightarrow n_A = \frac{13000}{2185} = \boxed{5.95}$$

With a minimum safety factor of 5.32, this gearbox design achieves the required performance parameters and should not fail. **The suggested gears have diametral pitch 10, face width 1.25”, pressure angle 20°, and are made of AGMA Cast Iron Grade 40. The pinion has 20 teeth, and the gear has 40 teeth.**

Summary of Power Train

In this memo, the motor and the gears were selected and analyzed based on the desired design specifications. Calculations involved determining the required power and torque of the motor to move the coffee beans at the specified speed. The values for motor power were then used to calculate the optimal gear selection for a high factor of safety. The properties of the 3” ACME power screw selected to move the pushers determined the required torque for the design. Careful selection of motors determined that Analysis of each gear in the power train was performed to check for both static and fatigue failure. The selected motor was motor 59825K49, which can deliver 1/8 hp at 475 rpm, meeting the required power and rotational speed needed to move the beans at 1 in/s or 60 in/min.

As for the gear train, we selected pinion 5172T21 and gear 5172T23. The estimated gear ratio required was 4, so two pinion and gear sets with gear ratios of 2 were enough. The chosen gears were then analyzed under the stress of the motor and the rest of the system. The gears were first assumed to have 180 BHN and 99% reliability. Using these assumptions, the factor of safety was only marginally sufficient. By increasing gear size and using pinion 5172T52 and gear 5172T54, the factor of safety able was increased without having to rely on more premium materials. The suggested gear material is AGMA Cast Iron Grade 40 with the pinion having 20 teeth and gear 40 teeth.

6 Works Cited

[1] M. Troy, "Glossary - sweet maria's Coffee Library," Sweet maria's, https://library.sweetmarias.com/glossary/ (accessed May 13, 2023).

[2] "SORTEX B - Optical sorting machine by Bühler GmbH | AgriExpo." https://www.agriexpo.online/prod/buehler-gmbh/product-168626-130113.html (accessed May 12, 2023).

[3] T. Chungcharoen, W. Limmun, W. Chanpaka, and N. Srisang, "Optimization of sorting efficiency of size grading machine using oscillating sieve with swing along width direction," *MATEC Web Conf.*, vol. 293, p. 04003, 2019, doi: 10.1051/matecconf/201929304003.

[4] "Concrete Mixers," *McNeilus*. https://www.mcneiluscompanies.com/concrete-mixers/ (accessed May 12, 2023).

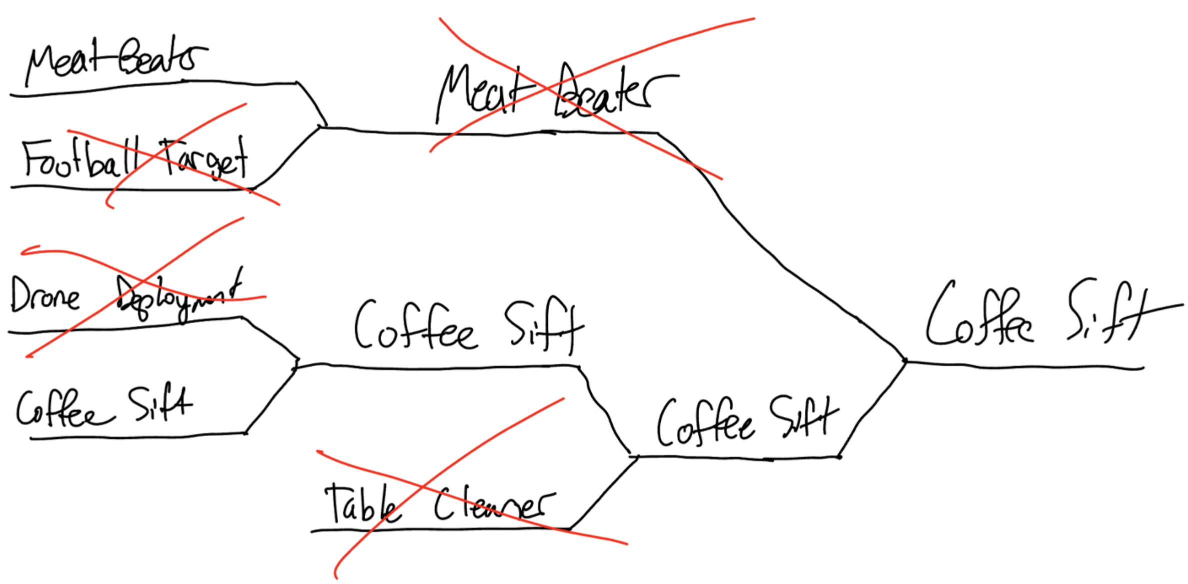
[5] "Archimedes' screw," *Wikipedia*. Apr. 30, 2023. Accessed: May 12, 2023. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Archimedes%27_screw&oldid=1152466093

7 APPENDIX

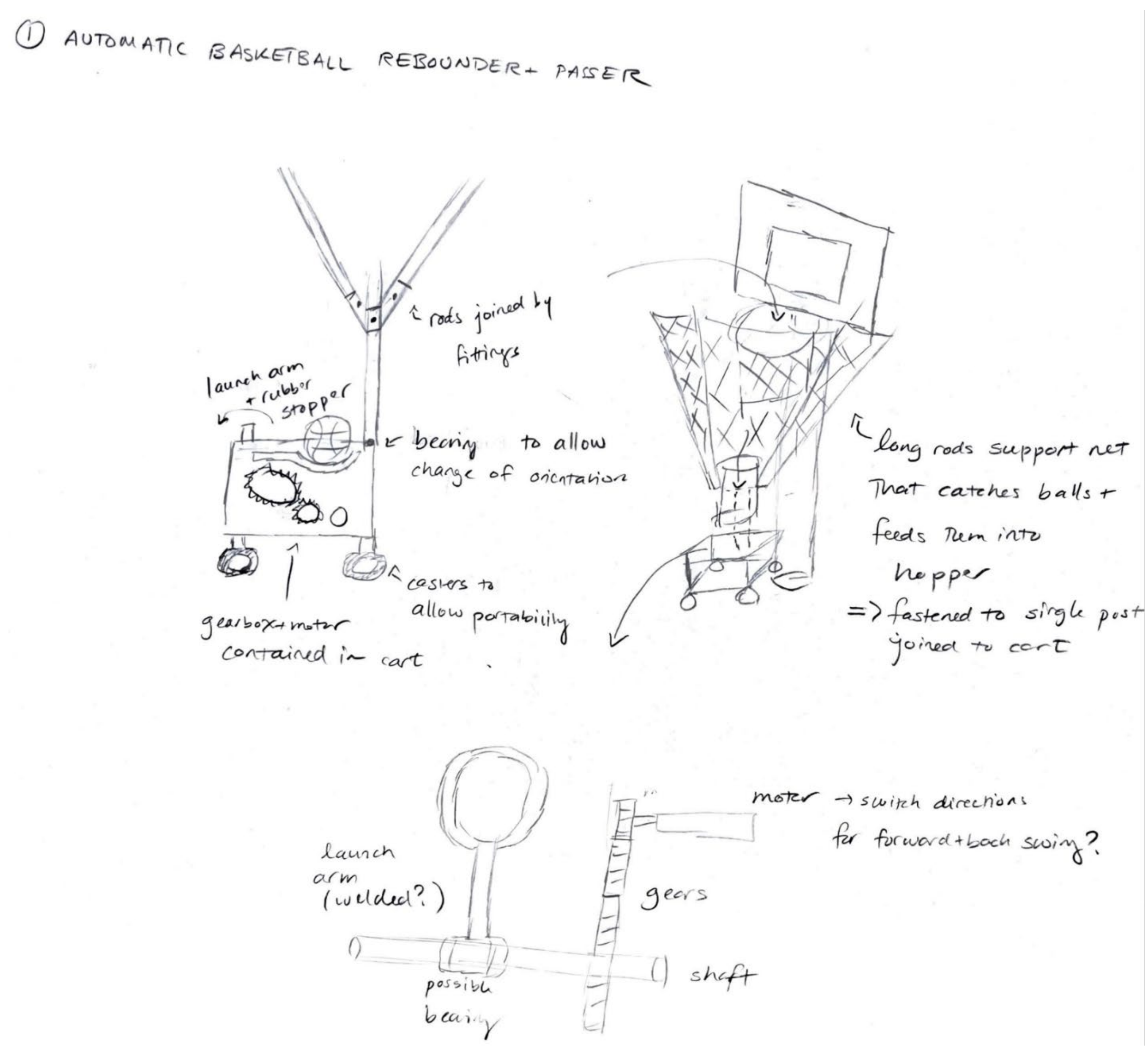
Each team member conceptualized and sketched five ideas they felt were novel, feasible, and met the requirements set by the scope of the project: each sketch contains a gearbox, powertrain, and distinct frame. The pages below contain these 30 sketches.

Team members took turns presenting their sketches and receiving feedback from the other members. Questions were asked to justify the use case and feasibility of each design. After all designs were presented, a tournament-style bracket was constructed with the top five ideas to determine the final idea for the proposal. This can be found below. These ideas were discussed within the group and down-selected based on the design specifications, customer needs, and competing products currently on the market. Potential issues, such as the design being too simple/complex for this course, were identified.

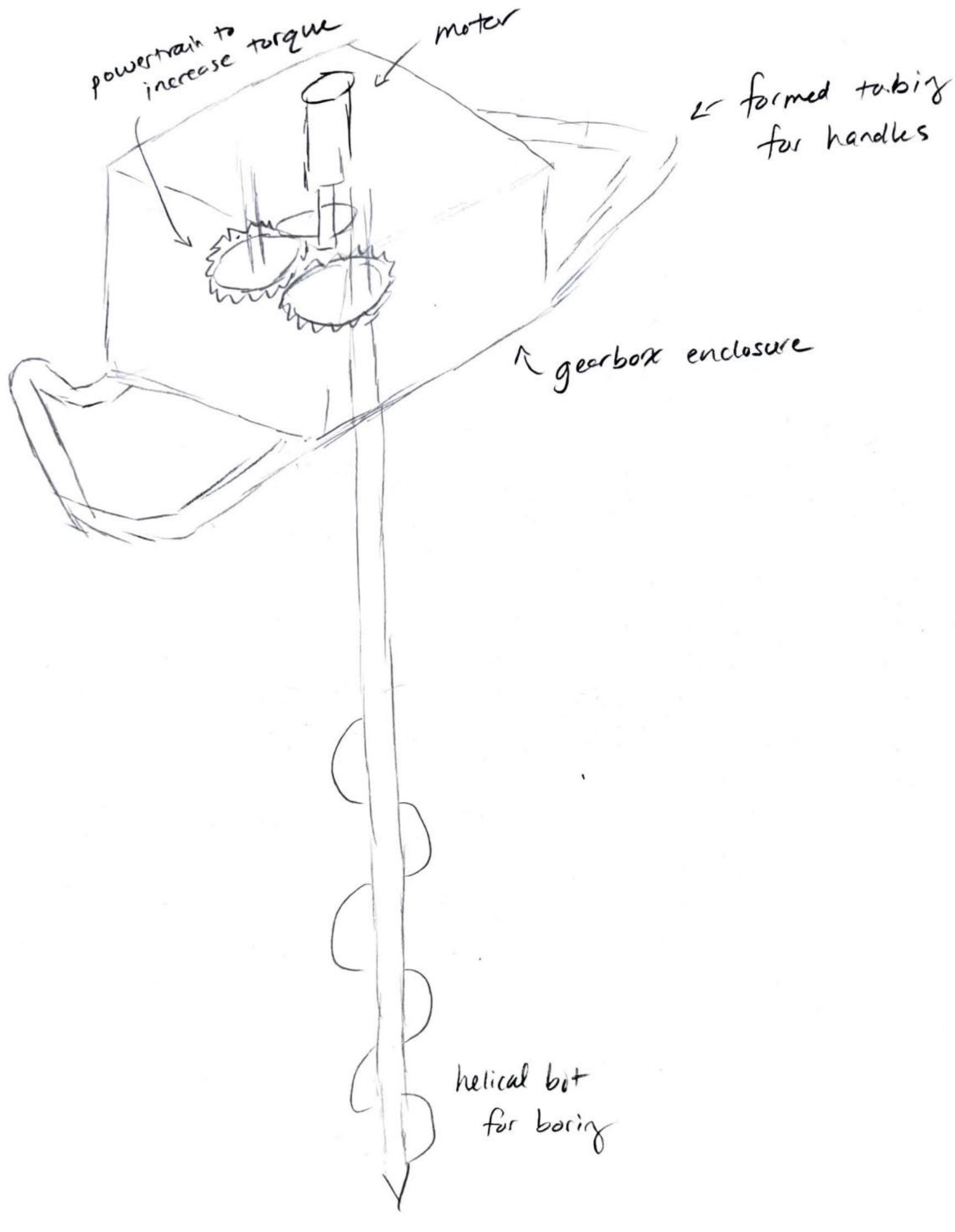
The final two designs for proposal were the automatic coffee sieve/sorter and the automatic chicken tenderizer. The group found both use cases exciting and the designs met all project requirements. However, the feasibility of the chicken tenderizer was questioned due to the dynamic nature of the device and the need for impact analysis. Professor Hashemi was consulted for his opinion. He felt the coffee sieve was more meaningful if the target market was a large enough supplier and the system would experience an appreciable load. Thus, the team proceeded with this idea for proposal. A detailed sketch can be found in Appendix A-2 below.



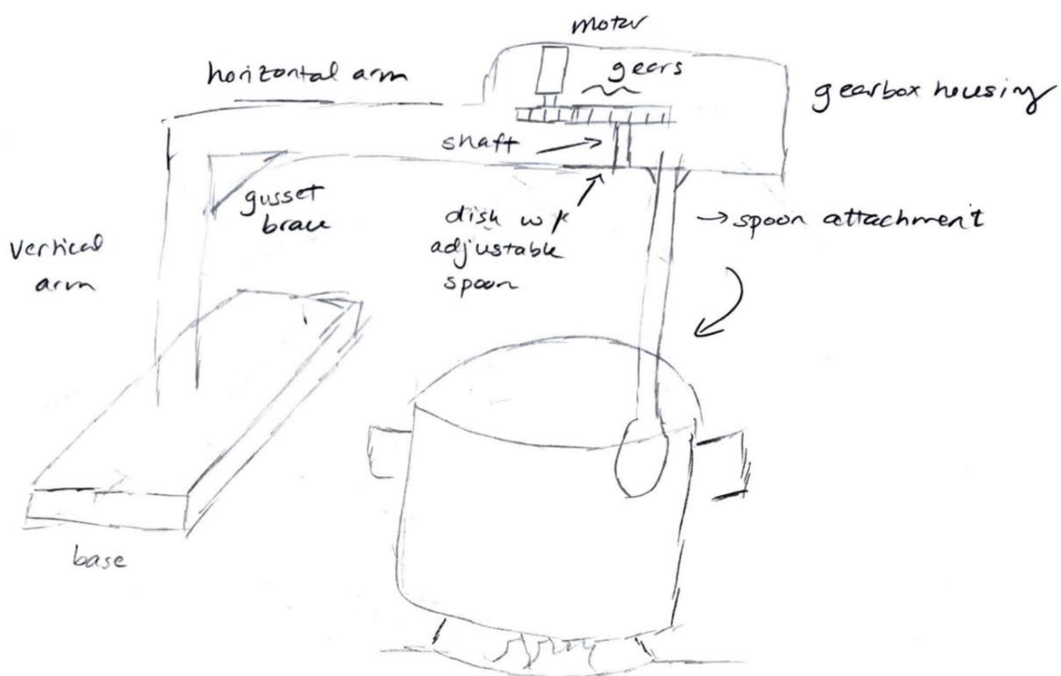
Appendix A-1: Nima Rakhshandeh Tali Sketches

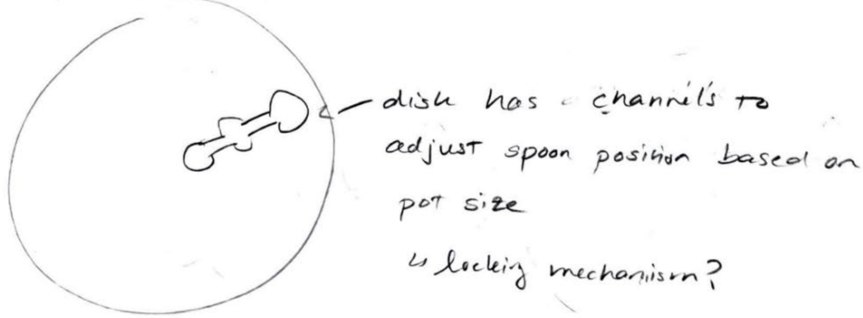


(2) AUGER FOR BORING HOLES IN SOIL &/OR ICE



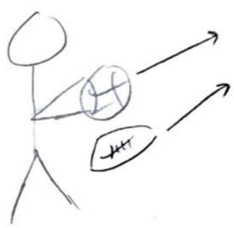
(3) AUTOMATIC POT STIRRER

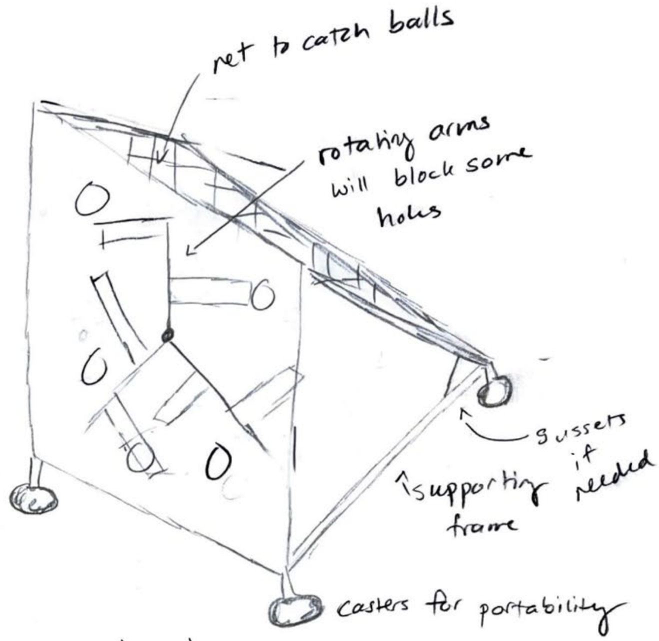




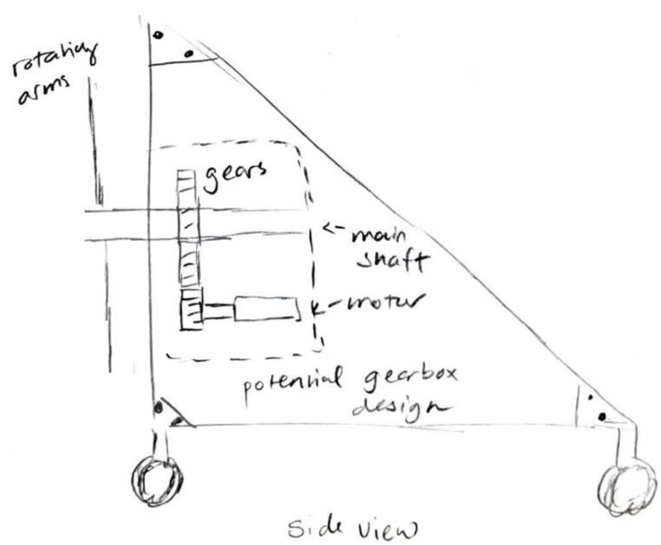
(4) TRAINER FOR PRECISION + TIMING OF BASKETBALL PASSES

+ OR FOOTBALL



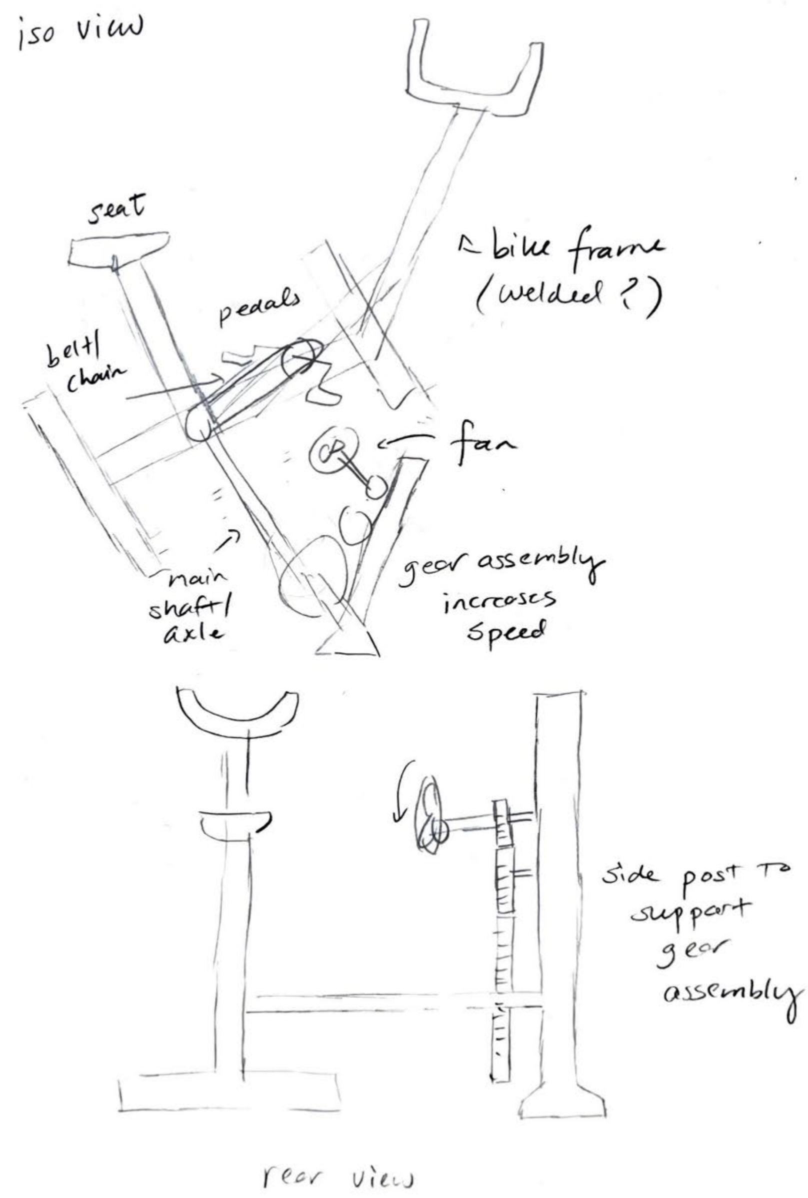


iso view

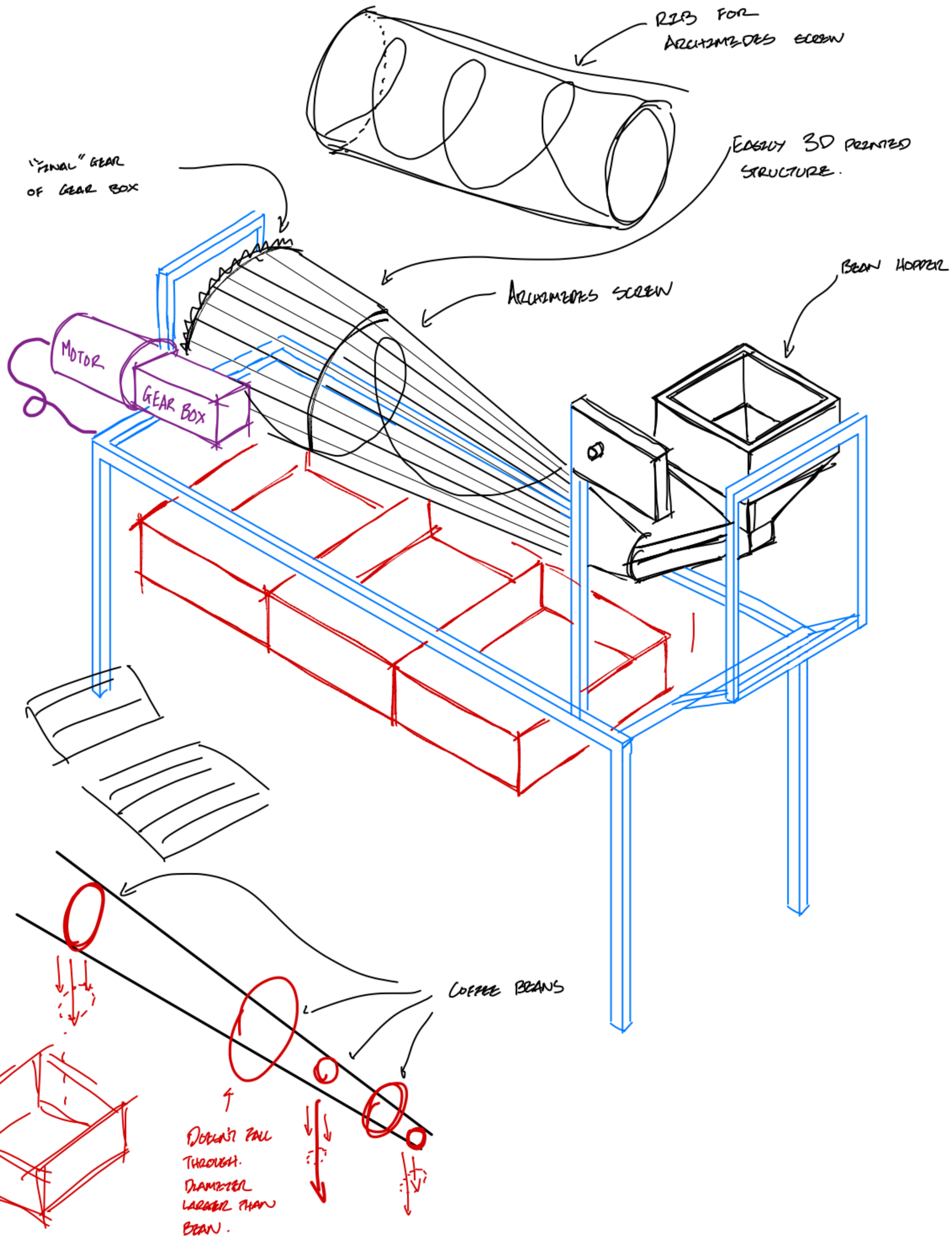


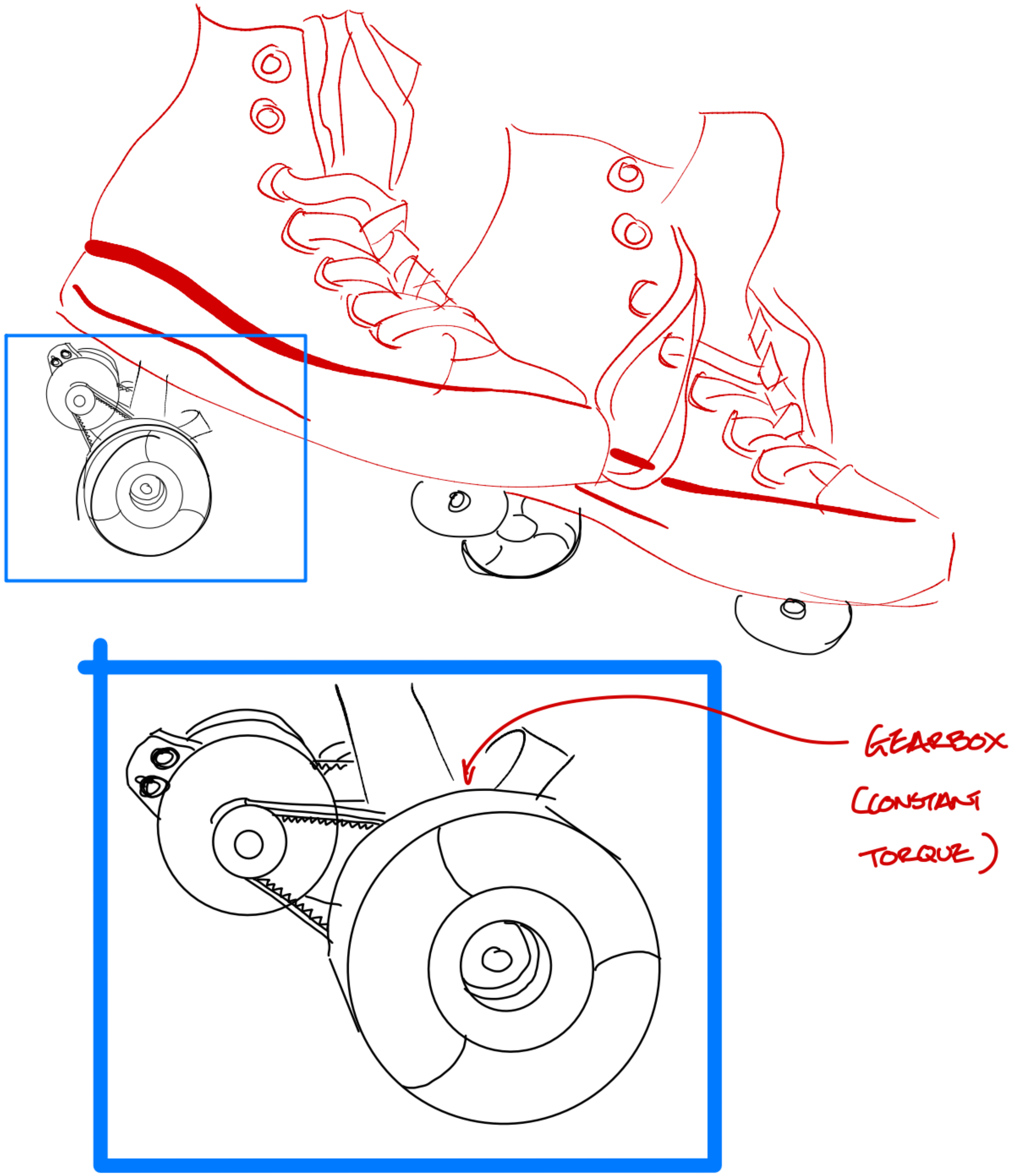
side view

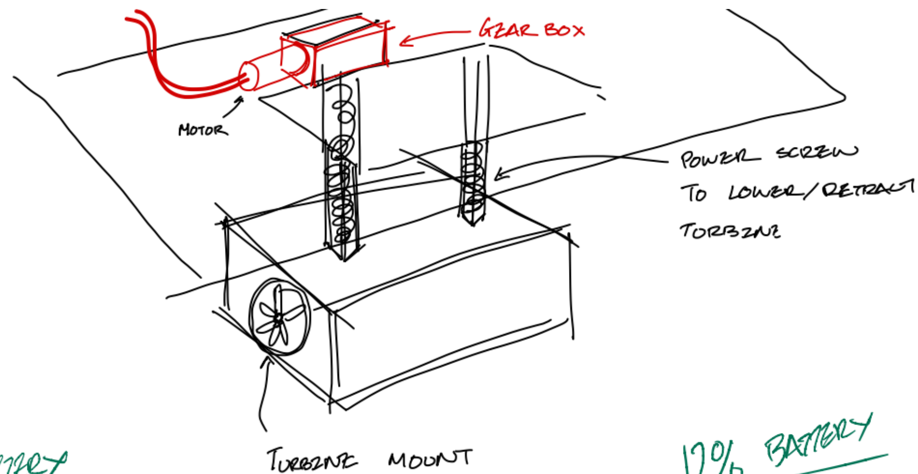
(5) OUTDOOR EXERCISE BIKE W/ SELF-POWERING FAN

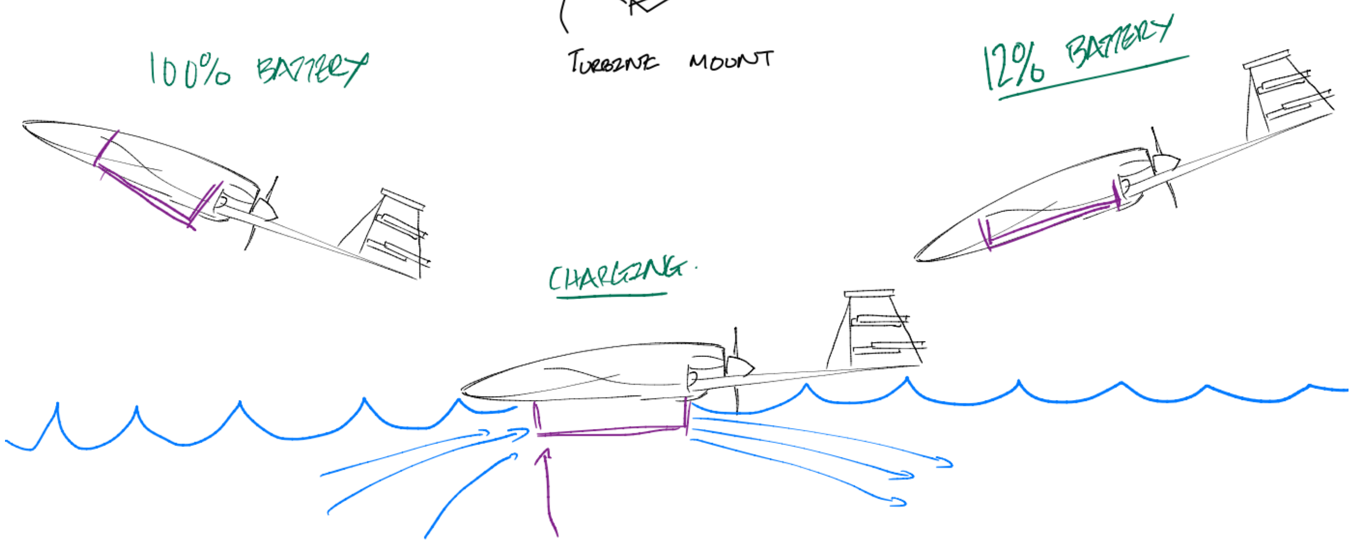


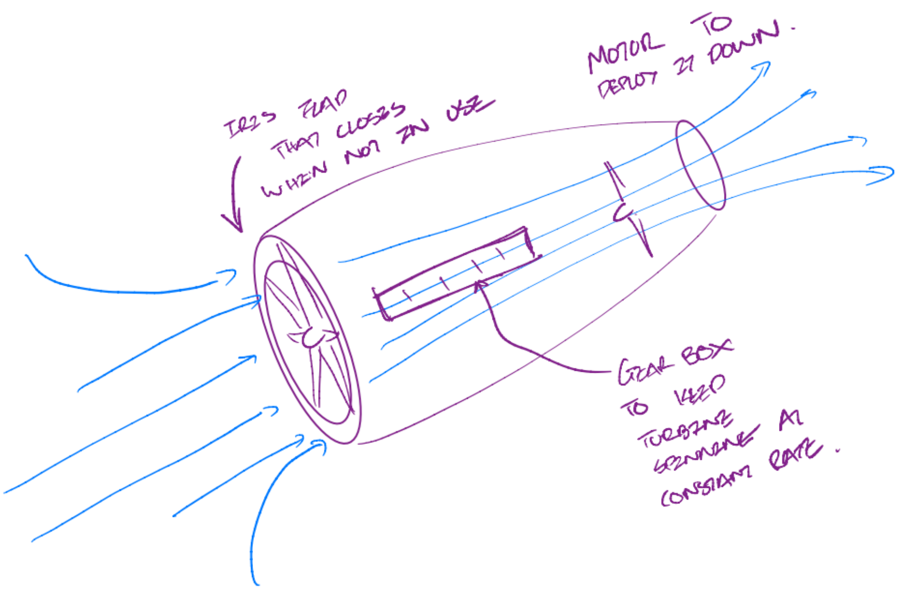
Appendix A-2: Samuel Scroggie Sketches



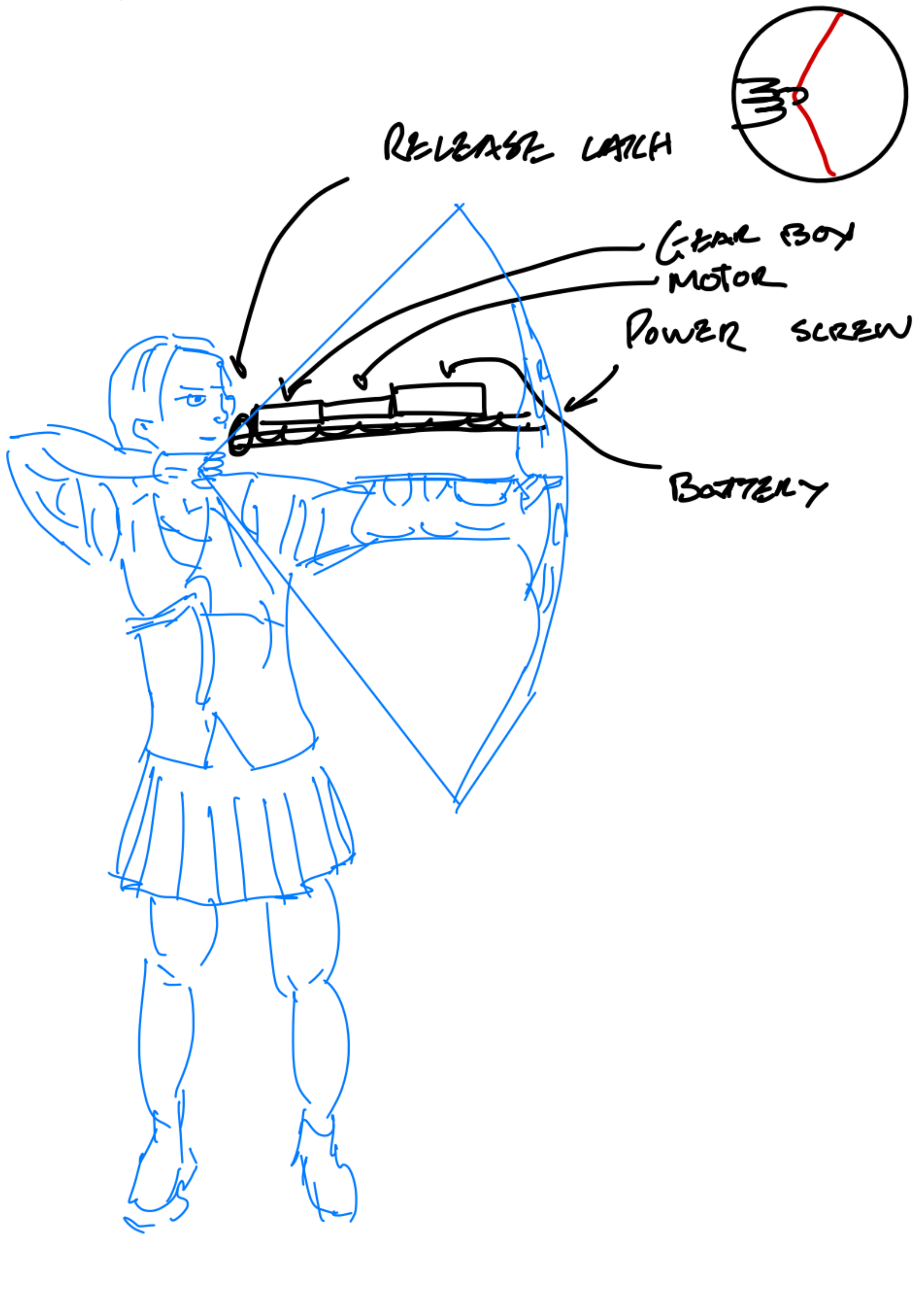

GEARBOX
(CONSTANT
TORQUE)


GEAR BOX
MOTOR
POWER SCREW
TO LOWER/RETRACT
TURBINE
TURBINE MOUNT


100% BATTERY
12% BATTERY
CHARGING.

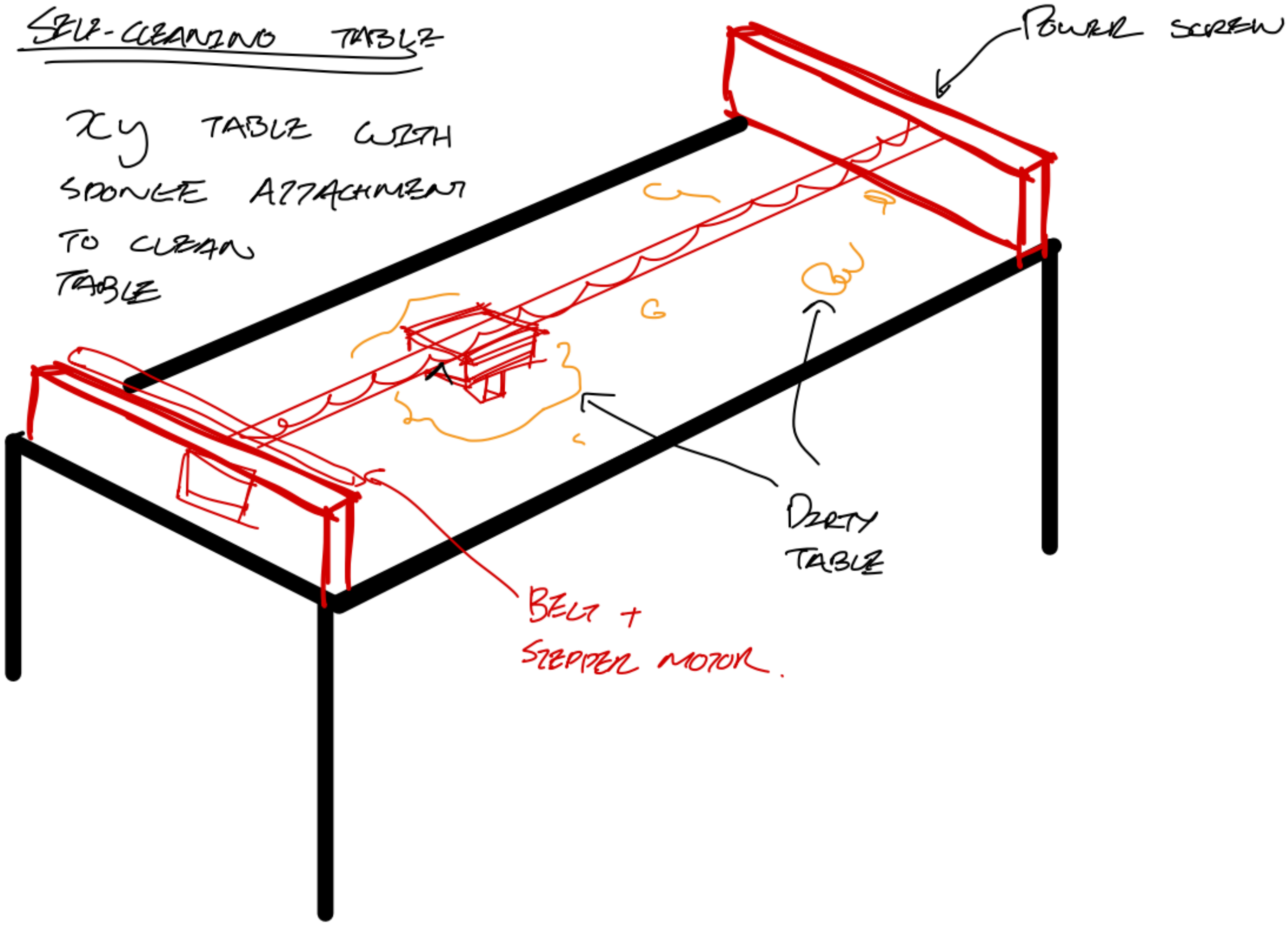

MOTOR TO
DEPLOY IT DOWN.
IRIS FLAP
THAT CLOSES
WHEN NOT IN USE
GEAR BOX
TO KEEP
TURBINE
SPINNING AT
CONSTANT RATE.

ARCHERY FOR DISABLED USERS



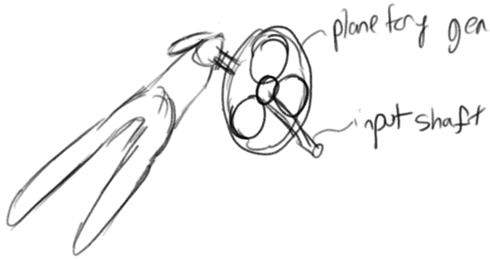
SELF-CLEANING TABLE

XY TABLE WITH SPONGE ATTACHMENT TO CLEAN TABLE



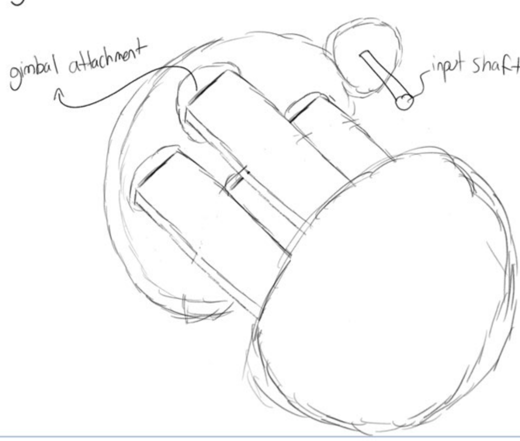
Appendix A-3: Joshua Weissert Sketches

Automated can opener

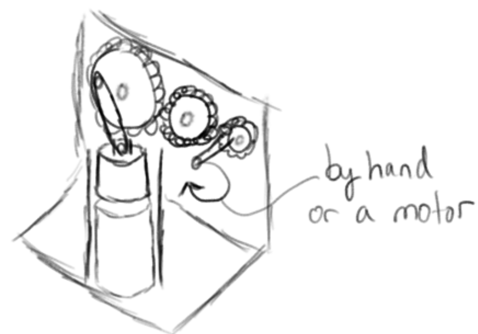


Leg stretcher

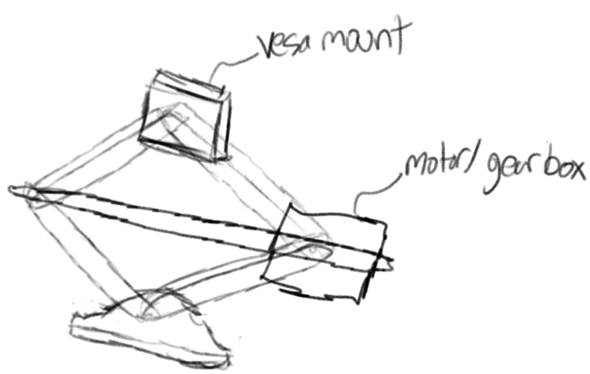
rotating shoe rack



Can Crusher

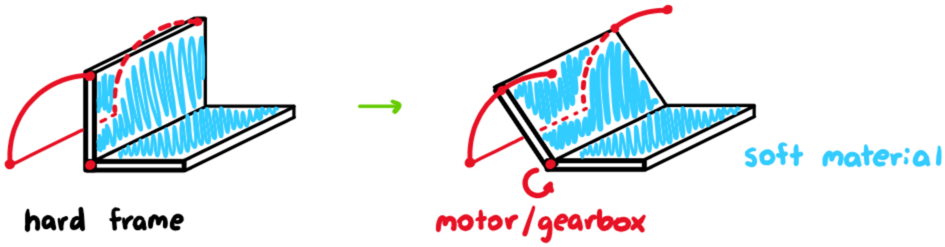


Raisable TV mount

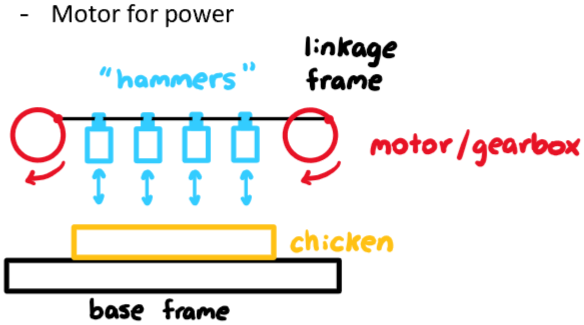


Appendix A-4: Jacob Ward Sketches

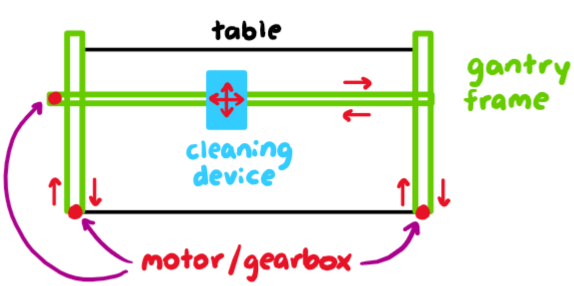
Sketch 1: Adjustable pillow
- Gearbox and motor to adjust back height
- Soft external frame for comfort
- Motor for power



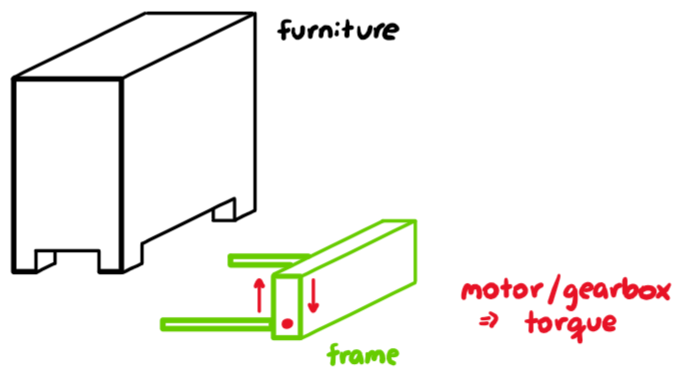
Sketch 2: Automatic Chicken Tenderizer
- Gearbox to control motors for pounding
- Food-safe frame
- Motor for power



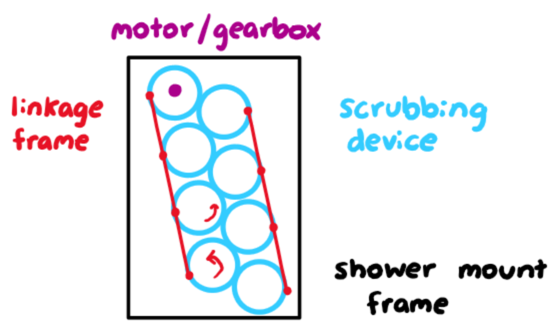
Sketch 3: Table Cleaner/Disinfector
- Gearbox to control motors
- xy-axis gantry frame to move
- Two motors to power each track



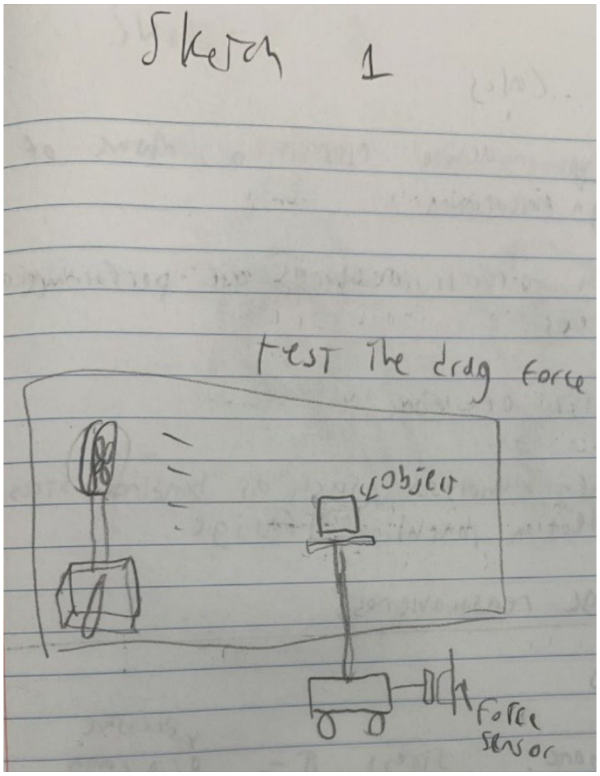
Sketch 4: Furniture Mover
- Gearbox to control motor
- Frame to allow for motion in the xy-plane
- Two motors to power movement

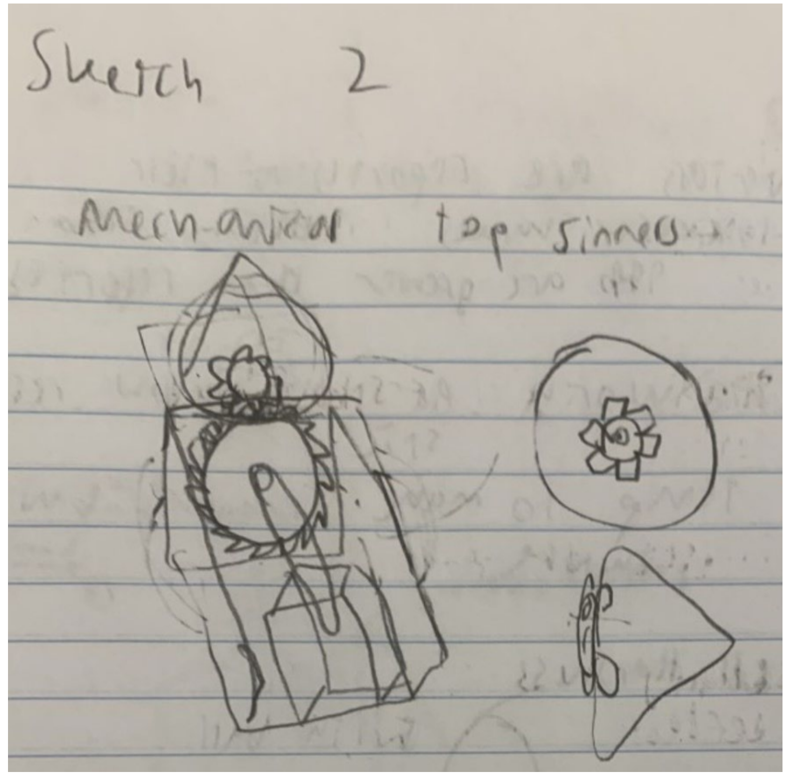


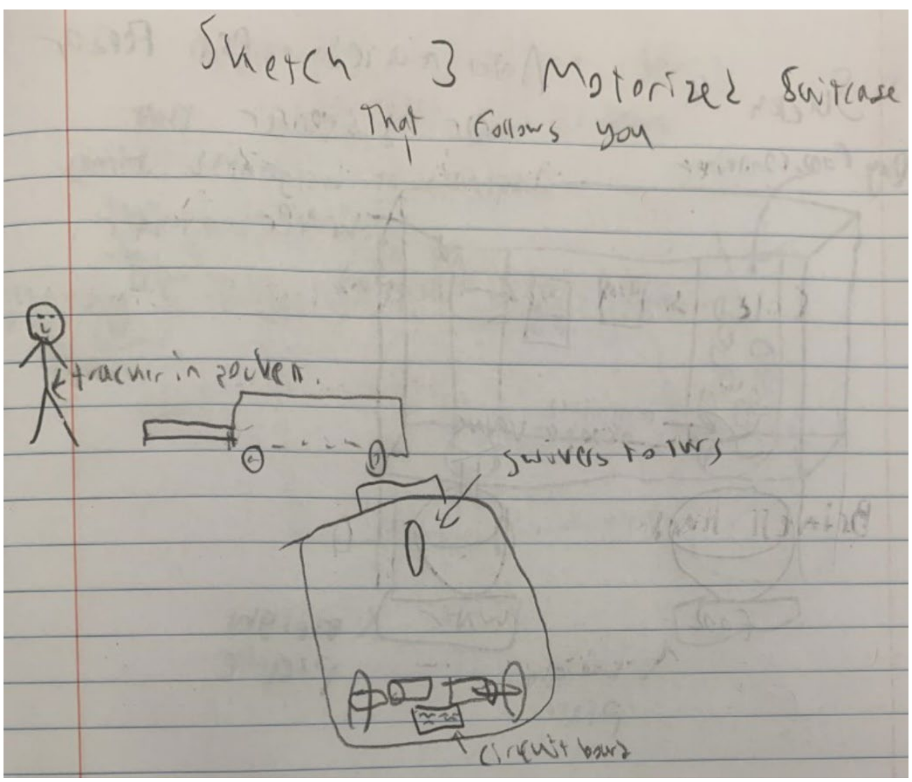
Sketch 5: Back Scrubber Machine
- Gearbox to control motors (torque vs. speed)
- Frame to mount in shower
- Motors to spin cleaning devices



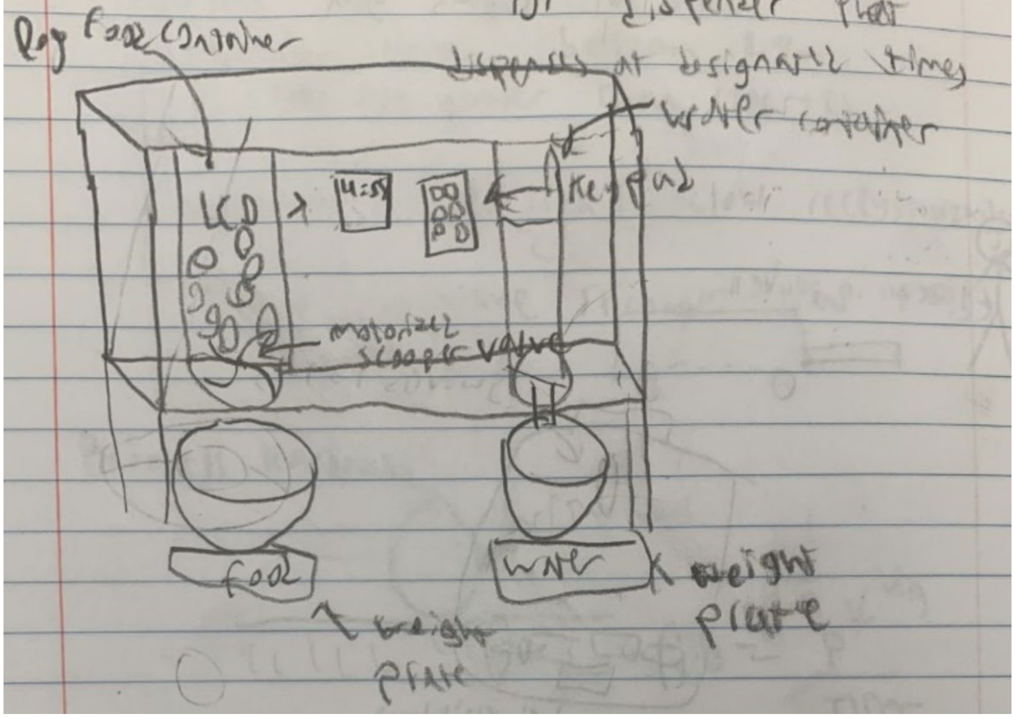
Appendix A-5: Sean Isaac Sketches



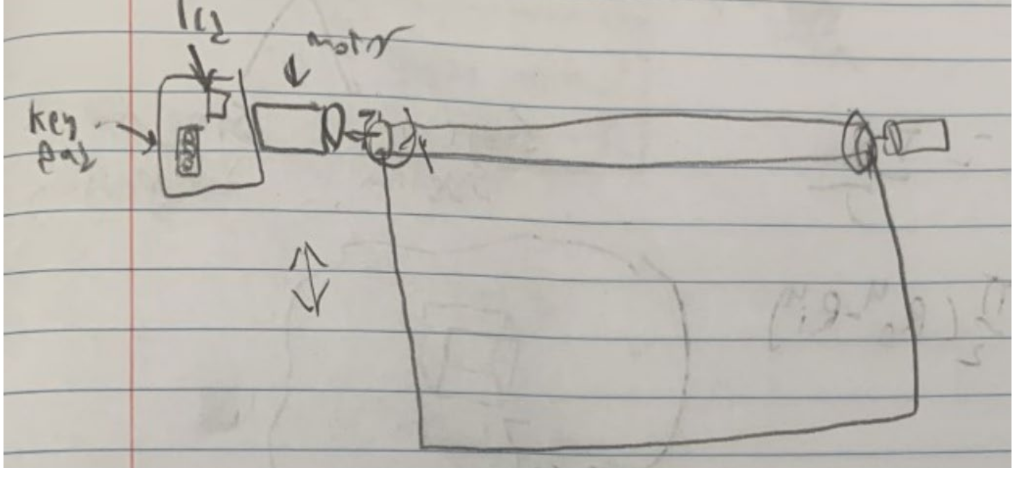




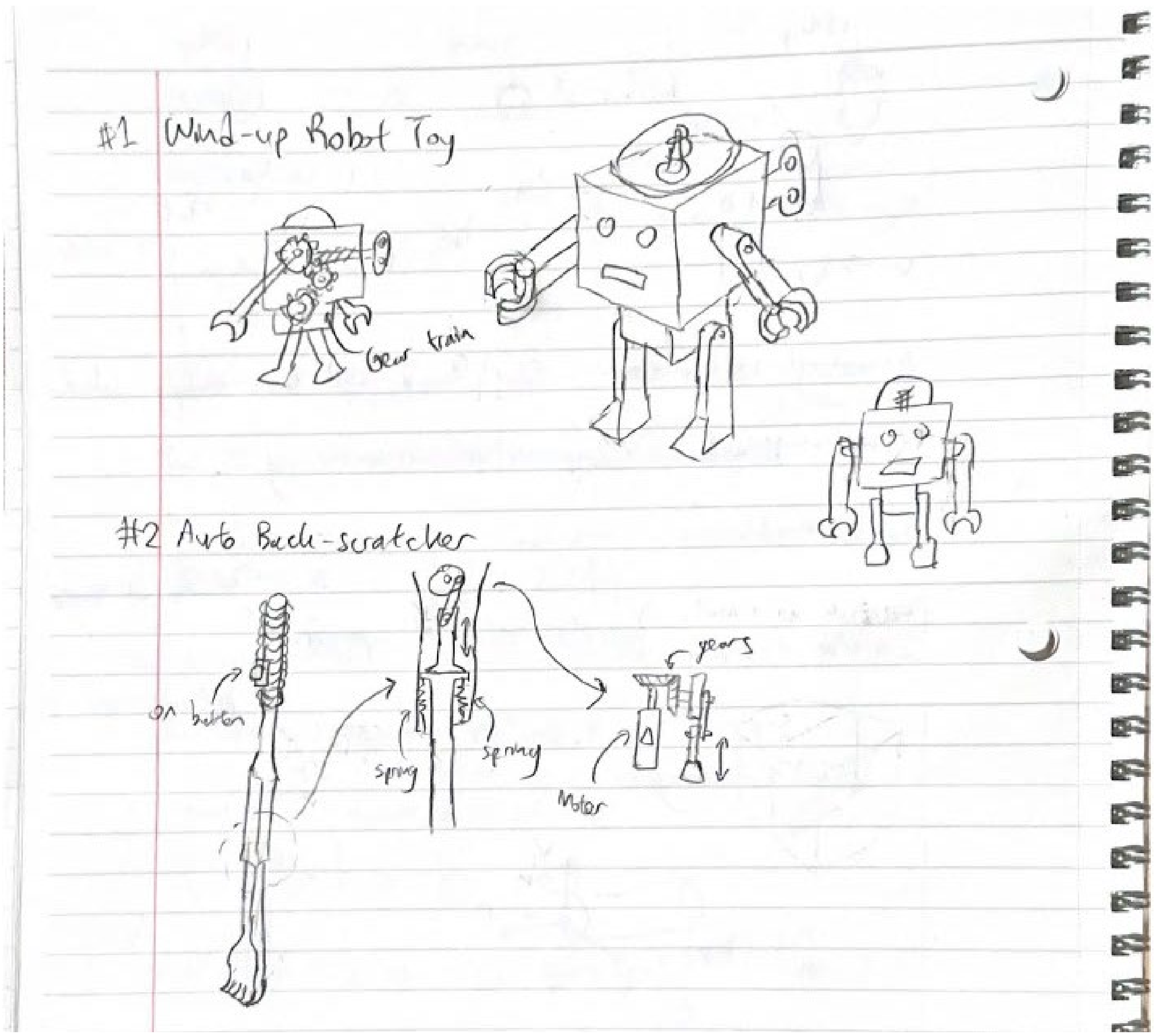
Sketch 4 Automated Pet Feeder or Dispenser that dispenses at designated times



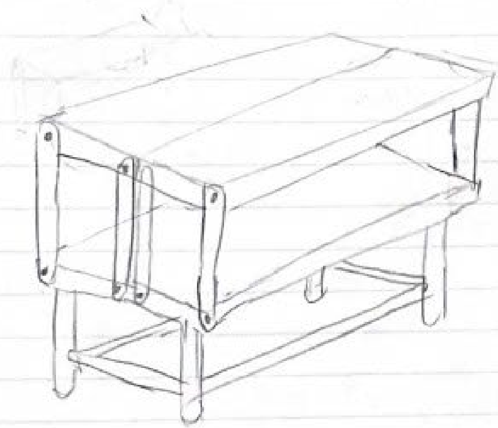
Sketch 5
Automated window blinds to open at set times

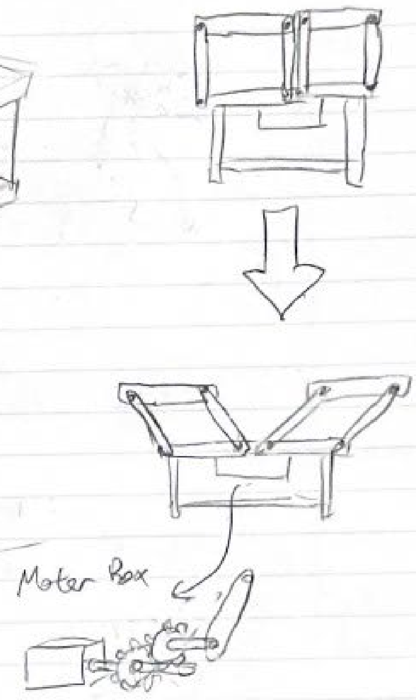


Appendix A-6: Christopher Zhang Sketches



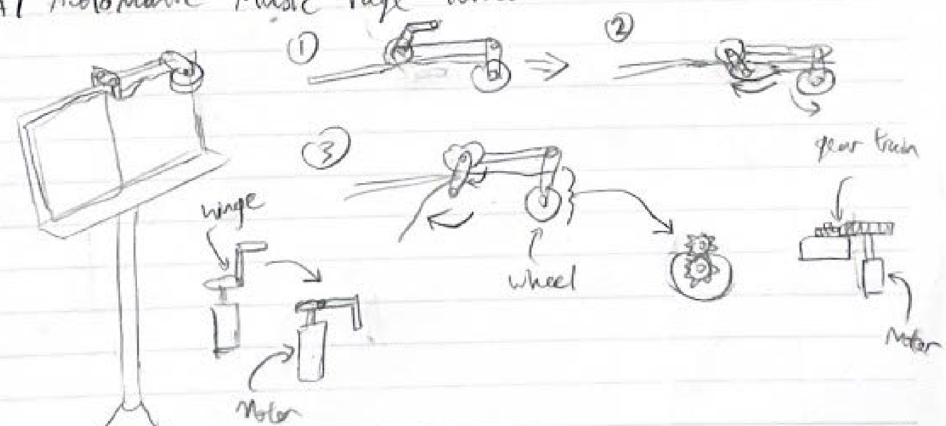
#3 Adjustable Display shelf





Locking system:

#4 Automatic Music Page Turner



#5 Automatic Meat Tenderizer