Coupled Electrical Oscillators

Joshua Weissert*
Northeastern University
(Dated: March 26, 2024)

Two LRC circuits can exchange energy through a coupling capacitor. A single LRC circuit and a pair of capacitively coupled LRC circuits were investigated and compared to a mechanical spring mass system. The two inductances were both found to be 99 ± 10 mH. The known inductances were said to be around 99 mH. When two LRC circuits are capacitevley coupled, a beat phenomenon occurs as energy is exchanged between the two circuits. Through data analysis, the beat frequency was found to be 1090 ± 150 Hz and the coupling capacitance was found to be 4.0 ± 0.4 nF. Compared to the measured capacitance value of 3.96 nF, the calculated quanitity is within error.

INTRODUCTION

An LRC circuit can be compared to a spring mass damping system where L is the inertia, R is the damper, and C is what stores energy. In this experiment, two LRC circuits will be coupled and a 555 chip will send a square wave into the system. Outputs of voltage across two capacitors will be studied to find inductor values, frequencies, and describe certain phenomenon.

Later in this experiment, two circuits with very similar capacitances and inductances will be coupled. When this happens, the frequencies of each circuit will be slightly different from each other and a "beat" phenomenon will occur. This is different from an acoustic beat where different frequencies add up. In a coupled electrical oscillator system, a beat is produced because of the transfer of energy between the two circuits. When one capacitor stars to discharge, the other capacitor will be charged from it. This exchange of charge and voltage creates a beat frequency different from the individual capacitor voltage frequencies. From this beat frequency, the coupling capacitor value can be calculated.

This experiment will study this coupled electrical oscillator system by finding different frequencies, inductances, capacitances, and resistance values and comparing them to expected values.

APPARATUS

The apparatus consisted of the following.

- Oscilloscope, Tektronix TBS 1052B
- Electrical Box with switches, connectors, inductors, and capacitors
- Multimeter, Extech
- Decade Capacitor, CDE CDA5
- MATLAB

Figure 1 shows the circuit used in this experiment. It shows the locations of capacitors, inductors, output pins, switches, and the 555 chip that outputs a square wave.

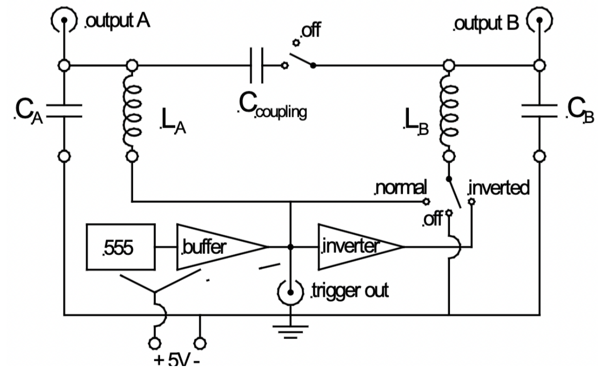

FIG. 1. Electrical Circuit for Coupled Electrical Oscillators

PROCEDURES AND RESULTS

A. Measure Inductance

Circuit A was set up using capacitor A (C_A) and inductor A (I_A). The capacitance of C_A was measured using the DMM to be 47.49 ± 0.01 nF. Circuit A was then hooked up to the oscilloscope and given power. The oscillations from output A in Figure 1 were observed and recorded. The oscillations where the excitation pulse goes to V=0 was analyzed as this has the largest number of cycles. The same set up was done using inductor B. A visual of this data along with curve fits is shown in Figure 2 and Figure 3.

By measuring one oscillation period from each inductor, the frequency could be measured. Then, using Eq. 1 [2], the inductor values were calculated. It is important to note that throughout this lab, whenever the inductors were powered on and measurements were being taken, the inductors were positioned to be away from each other and magnetic metals that could interfere with the inductance of the circuit.

$$\omega_0^2 = 1/LC \tag{1}$$

ω_0 is the frequency of the waveform, L is the circuits inductance, and C is the circuits capacitance. From this,

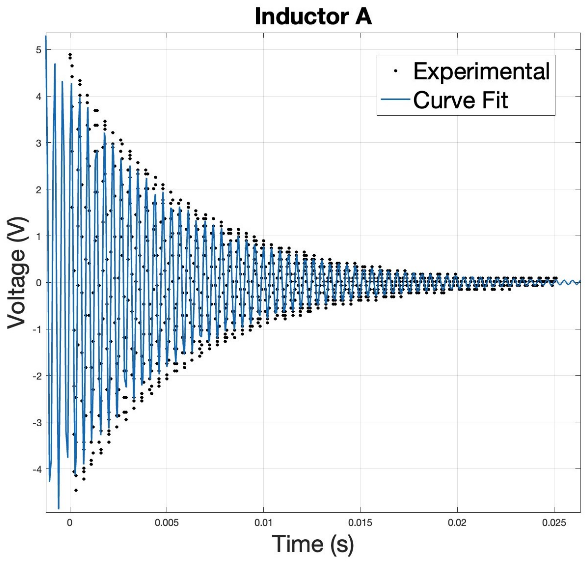

FIG. 2. Curve Fit for Inductor A Data

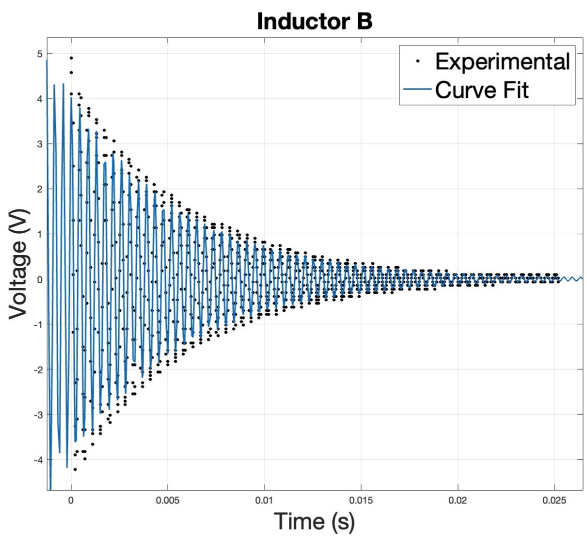

FIG. 3. Curve Fit for Inductor B Data

$L_A = 97.2 \pm 0.1$ mH and $L_B = 97.5 \pm 0.1$ mH. This shows that the two inductors have very similar inductances. One reason these errors are so small is the only error being taken into account is the time acquisition rate of the oscilloscope and the precision of the DMM, both of which are very small.

The data was fit to a curve in MATLAB according to the general equation for a decaying sinusoid shown in Eq. 2 [2].

$$y = Asin(\omega + C)e^{-\gamma x/2} \tag{2}$$

where A is the initial amplitude of the sinusoid, ω is the frequency, C is a phase shift, and γ is the damping coefficient which is related to the time constant of the exponent, τ, by $\tau = 2/d$. Once the damping coefficient is found, the resistance R of the circuit can be calculated using Eq. 3 [2].

TABLE I. Part A Values from Curve Fit and Given Equations

	A	σ for A	B	σ for B
ω' (Hz)	14560	5	14560	3
ω0 (Hz)	14560	359	14560	358
γ (Hz)	340.2	8.4	341.3	8.39
τ (ms)	5.88	0.15	5.86	0.14
L (mH)	99.33	4.90	99.33	4.88
R (Ohms)	33.79	1.86	33.90	1.86
Q	42.80	1.49	42.66	1.48

$$\gamma = R/L \tag{3}$$

The damping of the system reduces the frequency of the sinusoid as described in Eq. 4 [2].

$$\omega' = (\omega_0^2 - \gamma^2/4)^{1/2} \tag{4}$$

where ω' is the observed frequency of the system. The "quality factor" or Q-factor of the system is a dimensionless quantity defined by Eq. 5 [2].

$$Q = \omega_0/\gamma \tag{5}$$

Table I shows the values attained by MATLAB for frequencies, γ, τ, L, R, and Q for each circuit. The uncertainty values for these calculated L values are noticeably larger than the single period calculated L values. This is because of error propagation when calculating the frequency and error in the curve fitting software.

B. Compare Resistances

Using Eq. 3 the resistance of each circuit was calculated. As shown in Table I, R_A was found to be 33.79 Ω and R_B was 33.90 Ω. Using a DMM, the resistance of the inductors was measured to be 17.1 Ω. Using the physical dimensions of the inductor, a theoretical value was found to be 6.5 Ω but a small change in the measured values would put the actual values into the calculated range. The difference between the calculated R from the curve fitting and the measured R from the DMM can be explained by the added resistance of other electrical components in the circuit including the capacitors.

According to Eq. 4, the theoretical percent change that damping has on frequency is a 0.00738% decrease which is a minimal affect.

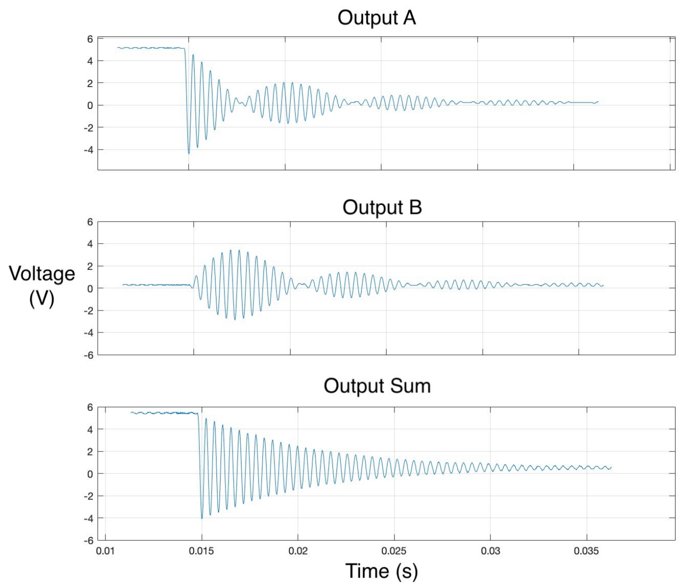

FIG. 4. Individual Channel Outputs and Their Sum for Coupled Electrical Oscillators

C. Independent Oscillators

The adjustable decade capacitor was connected to circuit B. The coupling switch was turned off with a "normal" excitation. Output A and B were observed on the oscilloscope. The variable capacitor was then slightly adjusted until both waveforms had the same period. The value of C_B for this to occur was 47.43 $\pm$ 0.01 nF compared to the C_A value of 47.49 $\pm$ 0.01 nF.

D. Unilateral Excitation of Coupled Oscillators

For this part of the experiments, the coupling capacitor was switched on and the B-circuit excitation was put to the middle position. The B-capacitor was finely adjusted to make sure the best minima was being achieved. The previous capacitor value found in part C was again, the best value to use for this.

Figure 4 shows the coupled oscillator output from channel A, channel B, and the sum of their outputs.

The output sum in this Figure looks extremely similar to the output from a single LRC circuit from part A. This shows that the energy in the system is being transferred between circuit A and circuit B at different points in time. This is similar to how a spring pendulum works. The motion will transition between two natural states. At some points the mass will be moving up and down but the pendulum motion is negligible and at other points the spring will not be changing length but the pendulum is moving back and forth. The energy is between transferred between two stable states.

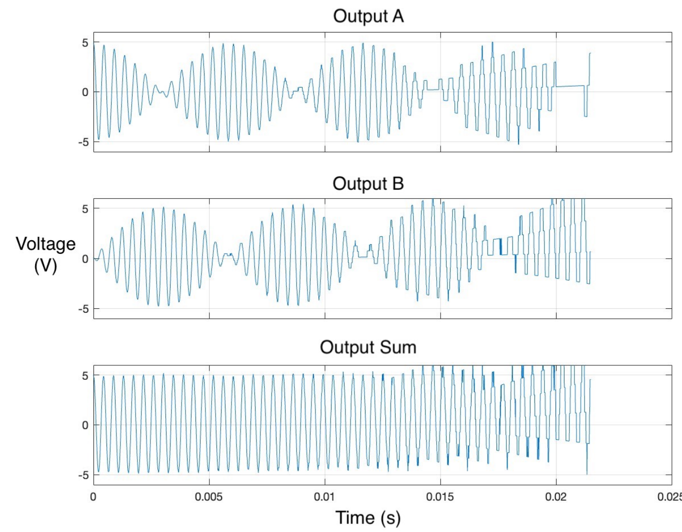

FIG. 5. Individual Channel Outputs and Their Sum for Coupled Electrical Oscillators without Decay

Figure 5 shows the coupled electrical oscillator when the decaying exponent term is divided out. As time goes on, the original voltage data points were getting smaller and smaller. This results in increased noise in the larger time values in the Figure. However, the output sum shows the conservation of energy in the system.

The "beat" frequency of this system can be found by finding the beat period. By picking two minimum, the beat frequency can be calculated. This gives us a ω_{beat} of 1087 $\pm$ 149 Hz. The uncertainty here was calculated by eyeballing the uncertainty when finding the minimum of the beat.

To curve fit the beat, the peaks of the channels would need to be located and using just the peaks, a curve could be fit to the equation of a generic sinusoid.

Using Eq. 6 [2], $C_{coupling}$ was calculated to be 3.98 nF. It was also measured directly by putting one end of the DMM into the L_A input and the other into the L_B input. This direct measurement gave a value of 3.96 nF which is very close to the theoretical value.

$$C_{coupling} = 0.5 * ((\frac{\sqrt{LC}}{\omega_{beat} * \sqrt{LC} - 1})^2 * \frac{1}{L} - C) \quad (6)$$

where L and C are the inductance and capacitance for both of the coupled circuits, respectively.

When the two uncoupled oscillators have slightly different frequencies, but are still coupled, this creates a beat phenomenon as seen in this experiment. The individual waveforms tranfer energy from one to the other, each creating their own beat with a similar frequency to the others.

It is also observed in this experiment that the rising edges of the excitation pulse last a shorter time and have less cycles than the falling edges.

SUMMARY

Throughout the experiment, different specifications of two LRC circuits were obtained and calculated using theory and curve fitting software. Part A showed that both circuit A and circuit B had the same properties as each other withing a single standard deviation. Part B showed that the resistance of the circuit comes from more than just the inductors. Part C showed that by giving slight adjustments using a decade capacitor, the two waveforms' periods could be synced up by eye. This is done to get a clear beat phenomenon. In Part D, this beat effect was studied. By looking at the individual voltage outputs of C_A and C_B, the individual beats can be clearly seen. Even more so when the decaying exponent was divided out of the data. From this, the transfer of energy between the two capacitors can be clearly seen. The beat frequency was also found by eyeballing the period to be $\omega_{beat} = 1087 \pm 149$ Hz.

All of our results were checked with the TA and were in the correct range. Some of the larger contributions to uncertainties include the curve fitting software used. Errors that were not taken into consideration during uncertainty calculation include the possible magnetic interference of the inductors with surrounding electromagnetic waves. While doing the experiment, it was noticed many times that the location of the inductors changed the oscilloscope output drastically because of this phenomenon. Final results of some important quantities are listed in Table II.

TABLE II. Measured and accepted values of specific electrical components and frequencies.

Quantity	Measured Value	σ	Accepted value	Deviation	Refs.
L_A (mH)	99.33	4.90	Approx. 99	1σ	[1]
L_B (mH)	99.33	4.88	Approx. 99	1σ	[1]
ω_{beat} (Hz)	1087	76	unknown	n/a	n/a
$C_{coupling}$ (nF)	3.98	0.20	unknown	n/a	n/a

* weissert.j@northeastern.edu

[1] Professor Haridas Kumurakuru
[2] H. Kumurakuru, B. Altunkaynak, D.Heiman, *"Coupled Electrical Oscillators"*